XVA Analysis From the Balance Sheet

Stéphane Crépey
https://math.maths.univ-evry.fr/crepey
LaMME, Univ Evry, CNRS, Université Paris-Saclay

Cours Bachelier, IHP, janvier–février 2018

Outline

- Since the 2008 crisis, investment banks charge to their clients, in the form of rebates with respect to the counterparty-risk-free value of financial derivatives, various add-ons meant to account for counterparty risk and its capital and funding implications.
- These add-ons are dubbed XVAs, where VA stands for valuation adjustment and X is a catch-all letter to be replaced by C for credit, D for debt, F for funding, M for margin, K for capital (!), and so on.

- Pricing XVA add-ons at trade level
  - funds transfer price (FTP)
- But also accounting XVA entries at the aggregate portfolio level
  - In June 2011 the Basel Committee reported that

    > *During the financial crisis, roughly two-thirds of losses attributed counterparty credit risk were due to CVA losses and only about o third were due to actual defaults*

  - In January 2014 JP Morgan has recorded a $1.5 billion FVA loss
- Individual FTP of a trade actually computed as portfolio incremental XVAs of the trade

- XVAs deeply affect the derivative pricing task by making it global, nonlinear, and entity-dependent
- But, before coming to the technical implications, the fundamental points are:
  1. To understand what deserves to be priced and what does not
     - $\supseteq$ Double counting issues
  2. To establish not only the pricing, but also the corresponding collateralization, accounting, and dividend policy of the bank

- Hull and White (2012) Risk Magazine notes, although one need not agree with their conclusions that rely on perfect market assumptions, had the merit to shift the debate to the right grounds:
  - Properly accounting for the misalignement of interest between the shareholders and creditors and the bank
  - And, the way we put it, for the coexistence and interaction within the bank of XVA versus “clean” desks.

References

- Own:
  - C. Albanese and S. Crépey. XVA Analysis From the Balance Sheet. *Working paper* 2017.
  - C. Albanese, S. Caenazzo and S. Crépey. Credit, Funding, Margin, and Capital Valuation Adjustments for Bilateral Portfolios. *Probability, Uncertainty and Quantitative Risk* 2017.

- Others:
  - On the funding issue,
    - Burgard and Kjaer (BK) FVA papers,
    - Andersen, L., D. Duffie, and Y. Song (2016). Funding value adjustments. ssrn.2746010;
  - On the capital issue,
    - The Solvency II actuarial literature,
    - Green and K enyon (GK) K VA papers.

The XVA benchmark model

- BK and GK papers (dozens of thousands of downloads on ssrn and arxiv!!) illustrate their points using a Black–Scholes model S for an underlying market risk factor, in conjunction with independent Poisson counterparties and bank defaults
- Warning: using a Black–Scholes (replication) framework as an XVA toy model is convenient.
- But, as FVA and KVA are mostly about market incompleteness in our opinion, there are some pitfalls to it.

The XVA benchmark model (cont'd)

- BK advocate a replication XVA approach and blame risk-neutral approaches outside the realm of replication (see the first paragraph in their 2013 paper).
- But BK papers themselves end-up doing what they call semi-replication, which is nothing but a form of risk-neutral pricing without (exact) replication.
- Addressing the KVA by replication, as done under the GK approach, is a bit of a contradiction in terms.
- More detailed comparison and comments in later sections of these slides.

Synopsis

Assumption 1

For accepting a new deal, shareholders need be at least indifferent given the cash flows before τ only.

- From this point of view, a key distinction is between
  - the cash flows received by the bank prior its default time τ
  - the cash flows received by the bank during the default resolution period starting at τ.
- The first stream of cash flows affects the bank shareholders, whereas the second stream of cash flows only affects creditors.

Assumption 2

A bank cannot replicate jump-to-default related cash-flows.

- Own default related cash-flows, in particular
- See Castagna and Fede (2013, Section 10.7)

1. Based on this market incompleteness tenet, we show that the all inclusive XVA add-on (funds transfer price FTP) that aligns derivative prices to the interest of bank shareholders is

$$\mathrm{FTP} = \mathrm{CVA} + \mathrm{FVA} + \mathrm{KVA},$$

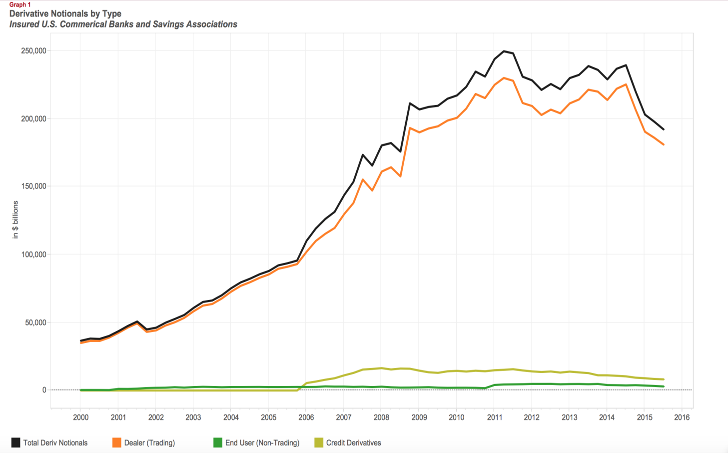
   a. to be contrasted with the "fair valuation" $(\mathrm{CVA} - \mathrm{DVA})$ of counterparty risk,
   b. where each XVA term is nonnegative (and solves a nonstandard backward SDE stopped before the bank default time τ).

université PARIS-SACLAY

2. Meant incrementally at every new deal, the above XVA add-on will be interpreted dynamically as the cost of the possibility for the bank to go into run-off,
   - i.e. lock its portfolio and let it amortize in the future,

   while staying in line with shareholder interest, from any point in time onward if wished.

→ Basel III Pillar 2 FTP as a "soft landing" or "anti-Ponzi" corrective pricing scheme accounting for counterparty risk incompleteness:



Contra-assets and Contra-liabilities

CA: Contra-assets, entail the valuation of all cash-flows related to the credit risk of either the counterparties or the bank and occurring before the default of the bank itself, i.e. having an impact on shareholder value.

- CVA, FVA, ...

CL: Contra-liabilities, entail the valuation of all the cash-flows received by the bank during the resolution process starting at its default time, i.e. only having an impact on bank creditors, by modifying the recovery rate of the bank, but not on shareholders.

- DVA, FDA, ...

Outline

- In this section we present the main ideas of our XVA approach in an elementary static one-year setup, with r set equal to 0.
- Assume that at time 0 a bank, with equity $\mathrm{E} = w_0$ corresponding to its initial wealth, enters a derivative position (or portfolio) with a client.
- Let $P = \mathbb{E}\mathcal{P}$ denote the mark-to-market of the deal ignoring counterparty risk and assuming risk-free funding.

- We assume that the bank and its client are both default prone with zero recovery.
- We denote by J and J_1 the survival indicators of the bank and its client at time 1
    - Both being assumed alive at time 0
    - With default probability of the bank $\mathbb{Q}(J = 0) = \gamma$
    - And no joint default for simplicity, i.e $\mathbb{Q}(J = J_1 = 0) = 0$.

XVA Cost of Capital Pricing Approach

- In order to focus on counterparty risk and XVAs, we assume that the market risk of the bank is perfectly hedged by means of perfectly collateralized back-to-back trades
    - The back-to-back hedged derivative portfolio reduces to its counterparty risk related cash flows
- At the bottom of this work lies the fact that a bank cannot replicate jump-to-default exposures
- Cost of capital pricing approach in incomplete counterparty risk markets

- Standing risk-neutral valuation measure $\mathbb{Q}$
- Derivative entry prices in our sense include, on top of the valuation of the corresponding cash-flows, a KVA risk premium
  - Risk margin (RM) in a Solvency II terminology
  - Computed assuming $\mathbb{P} = \mathbb{Q}$, as little of relevance can be said about the historical probability measure for XVA computations entailing projections over decades
  - The discrepancy between $\mathbb{P}$ and $\mathbb{Q}$ is left to model risk
- Cost of capital pricing approach applied to the counterparty risk embedded into the derivative portfolio of a bank

Cash Flows

- The counterparty risk related cash flows affecting the bank before its default are its counterparty default losses and funding expenditures, respectively denoted by $\mathcal{C}^\circ$ and $\mathcal{F}^\circ$.
- The bank wants to charge to its client an add-on, or obtain from its client a rebate,
  - depending on the bank being "seller or buyer",

  denoted by CA, accounting for its expected counterparty default losses and funding expenditures.

université PARIS-SACLAY

- Accounting for the to-be-determined add-on CA, in order to enter the position, the bank needs to borrow $(P-\mathrm{CA})^+$ unsecured or invest $(P-\mathrm{CA})^-$ risk-free, depending on the sign of $(P-\mathrm{CA})$, in order to pay $(P-\mathrm{CA})$ to its client.

- At time 1:
  - If alive (i.e. $J = 1$), then the bank closes the position while receiving $\mathcal{P}$ if its client is alive (i.e. $J_1 = 1$) or pays $\mathcal{P}^-$ if its client is in default (i.e. $J_1 = 0$).
    - Note $J_1\mathcal{P} - (1 - J_1)\mathcal{P}^- = \mathcal{P} - (1 - J_1)\mathcal{P}^+$. Hence the counterparty default loss of the bank appears as the random variable

$$\mathcal{C}^\circ = (1 - J_1)\mathcal{P}^+. \quad (1)$$

    In addition, the bank reimburses its funding debt $(P - \mathrm{CA})^+$ or receives back the amount $(P - \mathrm{CA})^-$ it had lent at time 0.
  - If in default (i.e. $J = 0$), then the bank receives back $\mathcal{P}^+$ on the derivative as well as the amount $(P - \mathrm{CA})^-$ it had lent at time 0.

- We assume that unsecured borrowing is fairly priced as $\gamma \times$ the amount borrowed by the bank, so that the funding expenditures of the bank amount to

$$\mathcal{F}^{\circ} = \gamma(P - \mathrm{CA})^{+},$$

deterministically in this one-period setup.

- We assume further that a fully collateralized back-to-back market hedge is set up by the bank in the form of a deal with a third party, with no entrance cost and a payoff to the bank $-(\mathcal{P} - P)$ at time 1, irrespective of the default status of the bank and the third party at time 1.

- Collecting cash flows, the wealth of the bank at time 1 is

$$\begin{aligned} w_1 &= \mathrm{E} - \mathcal{F}^\circ + (1-J)\big(\mathcal{P}^+ + (P-\mathrm{CA})^-\big) \\ &\quad + J\big(J_1\mathcal{P} - (1-J_1)\mathcal{P}^- - (P-\mathrm{CA})^+ + (P-\mathrm{CA})^-\big) - (\mathcal{P} - P) \\ &= \big(\mathrm{E} - (\mathcal{C}^\circ + \mathcal{F}^\circ - \mathrm{CA})\big) + (1-J)(\mathcal{P}^- + (P-\mathrm{CA})^+), \end{aligned} \tag{2}$$

  - as easily checked for each of the three possible values of the pair (J, J_1)

- The result of the bank over the year is

$$w_1 - w_0 = w_1 - \mathrm{E} = -(\mathcal{C}^\circ + \mathcal{F}^\circ - \mathrm{CA}) \\ + \underbrace{(1-J)\mathcal{P}^-}_{\mathcal{C}^\bullet} + \underbrace{(1-J)(P - \mathrm{CA})^+}_{\mathcal{F}^\bullet}.$$

- However, the cash flow $(\mathcal{C}^\bullet + \mathcal{F}^\bullet)$ is only received by the bank if it is in default at time 1, so that it only benefits bank creditors.
- Hence, the profit-and-loss of bank shareholders reduces to $-(\mathcal{C}^\circ + \mathcal{F}^\circ - \mathrm{CA})$, i.e. the trading loss-and-profit of the bank, which we denote by L, appears as

$$L = \mathcal{C}^\circ + \mathcal{F}^\circ - \mathrm{CA}. \tag{3}$$

Remark 1

- The derivation (2) allows for negative equity, which is interpreted as recapitalization.
- In a variant of the model excluding recapitalization, where the default of the bank would be modeled in a structural fashion as $\mathrm{E} - L < 0$ and negative equity is excluded, we would get instead of (2)

$$w_1 \quad = (\mathrm{E} - L)^+ + \mathbb{1}_{\{\mathrm{E}<L\}}(\mathcal{P}^- + (P - \mathrm{CA})^+). \tag{4}$$

- In our approach we consider a model with recapitalization for reasons explained later.

CVA and FVA

- In order to balance the trading loss and profit L, the bank charges to its client the add-on

$$\mathrm{CA} = \underbrace{\mathbb{E}\mathcal{C}^{\circ}}_{\mathrm{CVA}} + \underbrace{\mathbb{E}\mathcal{F}^{\circ}}_{\mathrm{FVA}}, \tag{5}$$

which accounts for the expected counterparty default losses and funding expenditures of the bank.

- Note that, since

$$\mathrm{FVA} = \mathbb{E}\mathcal{F}^{\circ} = \mathcal{F}^{\circ} = \gamma(P - \mathrm{CA})^{+}$$

(all deterministically in a one-period setup), (5) is in fact an equation for CA.

- Equivalently, we have the following semi-linear equation for $\mathrm{FVA} = \mathrm{CA} - \mathrm{CVA}$:

$$\mathrm{FVA} = \gamma(P - \mathrm{CVA} - \mathrm{FVA})^+,$$

which has the unique solution

$$\mathrm{FVA} = \frac{\gamma}{1+\gamma}(P - \mathrm{CVA})^+.$$

- Substituting this and (1) into (5), we obtain

$$\mathrm{CA} = \underbrace{\mathbb{E}[(1 - J_1)\mathcal{P}^+]}_{\mathrm{CVA}} + \underbrace{\frac{\gamma}{1+\gamma}(P - \mathrm{CVA})^+}_{\mathrm{FVA}}. \tag{6}$$

- Note that the realized recovery is

$$\mathcal{C}^\bullet + \mathcal{F}^\bullet = (1-J)(\mathcal{P}^- + (P - \mathrm{CA})^+)$$

because of the trade that occurred, but this was not anticipated and not reflected in the price of borrowing when the bank issued its funding debt.

- As the funding debt was fairly valued ignoring this, the value $\mathrm{FDA} = \mathbb{E}[(1-J)(P - \mathrm{CA})^+]$ of the default funding cash flow $\mathcal{F}^\bullet$ equals the cost $\mathrm{FVA} = \gamma(P - \mathrm{CA})^+$ of funding the position.

- But the FVA and the FDA do not impact the same economic agent, namely the FVA hits bank shareholders whereas the FDA benefits creditors.
- Hence, the net effect of funding is not nil to shareholders, but reduces to an FVA cost.

- In view of (3) and (5), observe that charging to the client a CA add-on corresponding to expected counterparty default losses and funding expenditures is equivalent to setting the add-on CA such that, in expectation, the trading loss-and-profit of bank shareholders is zero ($\mathbb{E}L = 0$), as it would also be the case without the deal.
- However, without the deal, the loss-and-profit of bank shareholders would be zero not only in expectation, but deterministically.

- Hence, to compensate shareholders for the risk on their equity triggered by the deal, under our cost of capital approach, the bank charges to its client an additional amount (risk margin)

$$\mathrm{KVA} = h\mathrm{E}, \tag{7}$$

where h is some hurdle rate, e.g. 10%.

- Moreover, since E can be interpreted as capital at risk earmarked to absorb the losses $(\mathcal{C}^\circ + \mathcal{F}^\circ)$ of the bank above CA, it is natural to size E by some risk measure of the bank shareholders loss-and-profit L.
- The all-inclusive XVA add-on to the entry price for the deal, which we call funds transfer price (FTP), is

$$\mathrm{FTP} = \underbrace{\mathrm{CA}}_{\text{Expected costs}} + \underbrace{\mathrm{KVA}}_{\text{Risk premium}} . \tag{8}$$

Monetizing the Contra-Liabilities?

- Let us now assume, for the sake of the argument, that the bank would be able to hedge its own jump-to-default risk through a further deal, whereby the bank would deliver a payment $(\mathcal{C}^\bullet + \mathcal{F}^\bullet)$ at time 1 in exchange of an upfront fee fairly valued as

$$\mathrm{CL} = \underbrace{\mathbb{E}\mathcal{C}^\bullet}_{\mathrm{DVA}} + \underbrace{\mathbb{E}\mathcal{F}^\bullet}_{\mathrm{FDA}=\gamma(P-\mathrm{CA})^+=\mathrm{FVA}} . \quad (9)$$

  - DVA and FDA stand for debt valuation adjustment and funding debt adjustment.

- Let CR denote the modified charge to be passed to the client when the hedge is assumed.
- Accouting for the hedging gain $\mathcal{H}^{cl} = \mathrm{CL} - (\mathcal{C}^{\bullet} + \mathcal{F}^{\bullet})$, the wealth of the bank at time 1 is now given by (cf. (2))

$$\begin{aligned} w_1 \quad &= (\mathrm{E} - (\mathcal{C}^{\circ} + \mathcal{F}^{\circ} - \mathrm{CR})) + \mathcal{C}^{\bullet} + \mathcal{F}^{\bullet} + \mathcal{H}^{cl} \\ &= \mathrm{E} - (\mathcal{C}^{\circ} + \mathcal{F}^{\circ} - \mathrm{CR}) + \mathrm{CL}. \end{aligned} \tag{10}$$

- By comparison with (2), the CL originating cash flow $(\mathcal{C}^\bullet + \mathcal{F}^\bullet)$ is hedged out and monetized as an amount CL received by the bank at time 0.
- The trading loss-and-profit of bank shareholders now appears as

$$L = w_0 - w_1 = \mathrm{E} - w_1 = \mathcal{C}^\circ + \mathcal{F}^\circ - \mathrm{CR} - \mathrm{CL}.$$

- The amount CR making L centered (and actually the same as before) is

$$\begin{aligned}\mathrm{CR} &= \mathbb{E}(\mathcal{C}^\circ + \mathcal{F}^\circ) - \mathrm{CL} \\ &= (\mathrm{CVA} + \mathrm{FVA}) - (\mathrm{DVA} + \mathrm{FDA}) = \mathrm{CVA} - \mathrm{DVA},\end{aligned} \tag{11}$$

  because FVA=FDA (cf. (9)).
- Hence, if the bank was able to hedge its own jump-to-default risk, in order to satisfy its shareholders in expectation, it would be enough for the bank to charge to its client an add-on $\mathrm{CR} = \mathrm{CVA} - \mathrm{DVA}$.

- The amount $\mathrm{CR} = \mathrm{CVA} - \mathrm{DVA}$ can be interpreted as the fair valuation of counterparty risk when market completeness and no trading restrictions are assumed (cf. Duffie and Huang (1996)).
- However, under our approach, in the present setup, the bank would still charge to its client a KVA add-on $h\mathrm{E}$ as risk compensation for the non flat loss-and-profit L triggered by the deal (with E sized by some risk measure of L).

Wealth Transfer Interpretation

- We see from (5) and (9) that CA can be viewed as the sum between CL and the fair valuation $\mathrm{CR} = \mathrm{CVA} - \mathrm{DVA}$ of counterparty risk.
- CL can be interpreted as an add-on that the bank needs to source from the client, on top of the fair valuation of counterparty risk, in order to compensate the loss of value to shareholders due to the inability of the bank to hedge its own jump-to-default risk.

- Due to this market incompleteness (or trading restriction), the deal triggers a wealth transfer from bank shareholders to creditors equal to CL, which then needs be sourced by the bank from its client in order to put shareholders back at value equilibrium in expected terms.

Funds Transfer Price Decomposition

- In conclusion, in a one-period setup, the FTP can be represented as

$$\begin{aligned}\mathrm{FTP} &= \underbrace{\mathrm{CVA} + \mathrm{FVA}}_{\text{Expected costs } \mathrm{CA}} + \underbrace{\mathrm{KVA}}_{\text{Risk premium}} \\ &= \underbrace{\mathrm{CVA} - \mathrm{DVA}}_{\text{Fair valuation } \mathrm{CR}} + \underbrace{\mathrm{DVA} + \mathrm{FDA}}_{\text{Wealth transfer } \mathrm{CL}} + \underbrace{\mathrm{KVA}}_{\text{Risk premium}} ,\end{aligned} \tag{12}$$

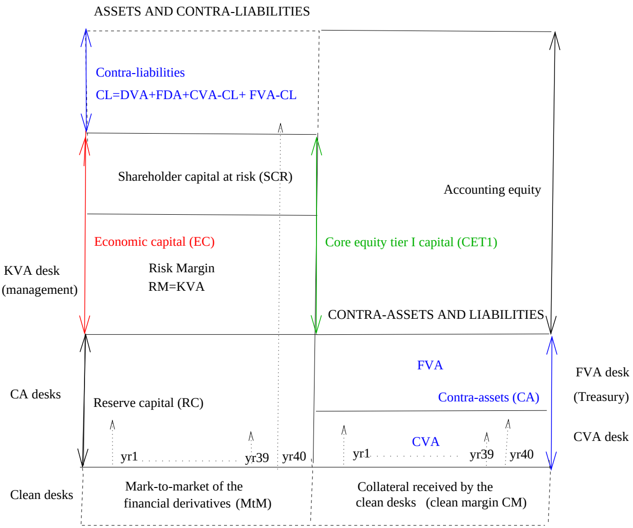
  where the CVA an the FVA are detailed in (6) and where the random variable L used to size the equity E in the KVA formula (7) is the bank shareholders loss-and-profit L as per (3).

Outline

- Need
  - of a multi-period model, which involves rebalancing between various banking accounts, for dealing with dynamic portfolios,
  - to cast the so-called contra-assets and contra-liabilities of the bank in a balance sheet perspective, in order to obtain not only the (entry) prices view on XVAs, but also the XVA accounting perspective

→ We introduce a capital structure model of a bank that shows the different bank accounts involved.

Balance Sheet of a Bank



A Bank With Three Floors

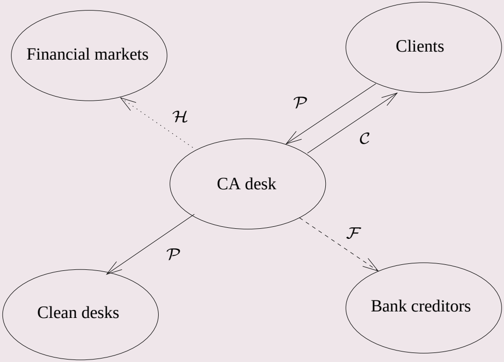
- We consider a central "CA desk" of the bank, in charge of absorbing counterparty default losses and funding expenditures.
- But not of the KVA, which is treated separately.

- The "$\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA}$ desk(s)" of the bank sells the corresponding "contra-assets" to the clients of the bank
    - Puts the ensuing "reserve capital" in an RC account, which is then used for absorbing the counterparty default losses and risky funding expenditures of the bank

- After the contracts have thus been cleaned of their counterparty risk and (other than risk-free) funding implications by the CA desk, the other trading desks of the bank, which we call clean desks (or "bottom floor") of the bank, are left with the management of the market risk of the contracts in their respective business lines, ignoring counterparty risk.

- The top (third) floor is the management in charge of the KVA payments, i.e. of the dividend distribution policy of the bank
    - Puts the “risk margin” sourced from the clients in an RM account, which is then gradually released to shareholders as a remuneration for their capital at risk

CA desk cash flows graph: contractually promised $\mathcal{P}$, risky funding $\mathcal{F}$, and hedging $\mathcal{H}$ cash flows



- In fact, we deal with two portfolios, the client portfolio between the clients of the bank and the CA desk and the cleaned portfolio between the CA desk and the clean desks.
- The corresponding (cumulative streams of) contractually promised cash flows are the same, denoted by $\mathcal{P}$. But, as intuitively clear and detailed in the sequel, counterparty risk only really impacts the client portfolio.

Another view on "mark-to-model":

- The RM account is continuously reset by the management of the bank to its (to be determined) theoretical target KVA level.
  - $(-d\mathrm{KVA}_t)$ amounts continuously flow from the RM account to the shareholder dividend stream
- The RC account is continuously reset by the CA desk to its (to be detailed) theoretical target $\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA}$ level
  - $(-d\mathrm{CA}_t)$ amounts continuously flow from the RC account to the shareholder dividend stream

- The CM account is continuously reset by the clean desks of the bank to the MtM value of the portfolio (ignoring counterparty risk, which they do not even "see" in their setup)
  - $(-d\mathrm{MtM}_t)$ amounts continuously flow from the CM account to the clean desks trading loss and profit process

→ Balance conditions

$$\mathrm{CM} = \mathrm{MtM}\,,\;\; \mathrm{RC} = \mathrm{CA} = \mathrm{CVA} + \mathrm{FVA}\,,\;\; \mathrm{RM} = \mathrm{KVA} \tag{13}$$

Assumption 3

The activity of each (clean or CA) trading desk, hence of the bank as a whole, is self-financing.

- At this stage this is only a broad axiom, already implicitly postulated in the one-period setup (cf. (2)), by which we mean that the wealth of a trading desk at a later time results from the development of its wealth at an earlier time as the sole consequence of its pricing, hedging, funding, and collateralization policy, and of the evolution of the underlying risk factors, without any further creation or annihilation of cash flows.

- Instead of viewing losses as money flowing away from the balance sheet, we view them as money flowing into it as refill, i.e. replenishment of the different bank accounts at their theoretical target level, until the point of default where the payers cease willing to do so.

- When this happens is modeled as a totally unpredictable time τ calibrated to the bank CDS spread, which we view as the most reliable and informative credit data regarding anticipations of markets participants about future recapitalization, government intervention and other bank failure resolution policies.

Comparison with the Merton model

- In a Merton mindset, the default of the bank in our setup would be modeled as the first time when the core equity (CET1) of the bank becomes negative, where CET1 depletions correspond to the bank trading losses L
- In the case of a bank, given recapitalisation and managerial resolution schemes, CET1 is constantly "refilled" by the shareholders and it is more realistic to model the default as a totally unpredictable (liquidity or operational) event at some exogenous time τ with intensity γ calibrated to the bank CDS spread.
  - cf. Duffie (2010)'s analysis of major bank defaults during the crisis

Comparison with the Merton model (Cont'd)

- The purpose of our capital structure model of the bank is not to model the default of the bank as the point of negative equity, which would be unrealistic (and in fact never occurs in our setup)
- Instead, our aim is to put in a balance sheet perspective the contra-assets and contra-liabilities of the bank, items which are not present in the Merton model.

Invariance Valuation Setup

- The clean desks of the bank, who are immunized against counterparty risk through the action of the CA desk, typically ignore the default of the bank in their modeling.
- They price and hedge in order to be non arbitrable ignoring the default of the bank, using some reference filtration $\mathbb{F}$ such that τ is not an $\mathbb{F}$ stopping time.

- But the bank is defaultable, hence the full model information used by the CA desk, as well as by the management of the bank in charge of the KVA payments, is a larger filtration $\mathbb{G}$ such that τ is a $\mathbb{G}$ stopping time.

Assumption 4

Any $\mathbb{G}$ stopping time η admits an $\mathbb{F}$ stopping time η' such that $\eta \wedge \tau = \eta' \wedge \tau$; any $\mathbb{G}$ semimartingale Y admits a unique $\mathbb{F}$ semimartingale Y', called the reduction of Y, that coincides with Y before τ.

- We denote by $J = \mathbb{1}_{[0,\tau)}$ the survival indicator process of the bank.
- For any left-limited process Y, we denote by

$$Y^{\circ} = Y^{\tau-} = JY + (1-J)Y_{\tau-}$$

the process Y stopped before time τ and we write $Y^{\bullet} = Y^{\circ} - Y$.

- Accounting for the bank default time τ, the time horizon of the model is $\bar{\tau} = \tau \wedge T$, where T is the final maturity of all claims in the portfolio, (first) assumed held on a run-off basis.

Definition 1

- By trading loss L^{cl} of the clean desks, we mean the negative of their wealth process $\mathcal{W}^{cl}$, as the latter results from their trading by an application of the self-financing assumption defined with respect to the reference filtration $\mathbb{F}$.
- By trading loss L^{ca} of the CA desk, we mean the negative of its wealth process $\mathcal{W}^{ca}$, as the latter results from its trading by an application of the self-financing assumption with respect to the filtration $\mathbb{G}$, stopped before τ for alignment with shareholder interest. That is,

$$L^{ca} = -(\mathcal{W}^{ca})^{\circ}. \tag{14}$$

Definition 1 (Cont'd)

- By trading loss L of the bank as a whole, we mean

$$L = -(\mathcal{W}^{cl} + \mathcal{W}^{ca})^{\circ} = (L^{cl})^{\circ} + L^{ca}. \quad (15)$$

- Consistency of valuation across the perspectives of the different desks of the bank is granted by the following:

Assumption 5

- Clean desks and CA desks generate trading losses that are martingales with respect to their respective pricing bases $(\mathbb{F}, \mathbb{P})$ and $(\mathbb{G}, \mathbb{Q})$,
- where the latter are such that $(\mathbb{F}, \mathbb{P})$ local martingales stopped before τ are $(\mathbb{G}, \mathbb{Q})$ local martingales,
- whereas the reductions of $(\mathbb{G}, \mathbb{Q})$ local martingales stopped before τ are $(\mathbb{F}, \mathbb{P})$ local martingales.

université PARIS-SACLAY

i.e. τ is an invariance time as per Crépey and Song (2017a).

- Standard (but by no means limitative) situation: immersion setup where $(\mathbb{F}, \mathbb{Q})$ local martingales are $(\mathbb{G}, \mathbb{Q})$ local martingales without jump at τ, in which case τ is an invariance time with $\mathbb{P} = \mathbb{Q}$.

As an immediate consequence of Assumption 5:

Corollary 1

The trading loss L^{cl} of the clean desks is an $(\mathbb{F}, \mathbb{P})$ local martingale. The trading losses L^{ca} and $L = L^{ca} + (L^{cl})^{\circ}$ of the CA desk and of the bank as a whole are $(\mathbb{G}, \mathbb{Q})$ local martingales without jump at time τ, their reductions are $(\mathbb{F}, \mathbb{P})$ local martingales.

Assumption 6

The bank cannot hedge its own jump-to-default exposure, hence $\mathcal{H} = \mathcal{H}^{\circ}$.

Given Y representing a process of cumulative cash flows or trading or hedging losses, respectively an XVA process, we denote by $\widetilde{Y}$ the corresponding process of cumulative OIS discounted cash flows or trading or hedging losses, respectively the corresponding OIS discounted XVA process.

Example 1

$\widetilde{L} = \int_0^{\cdot} \beta_t dL_t$, $\widetilde{\mathrm{CVA}} = \beta \mathrm{CVA}$.

Corollary 2

In the case of a cumulative cash flow or loss process Y, the process Y is a local martingale if and only if $\widetilde{Y}$ is a localmartingale.

Loss and Dividend Processes

Lemma 1

The trading loss processes L^{ca} of the CA desk and L of the bank as a whole, both risk neutral local martingales by application of Corollaries 1 and 2, are given by

$$\begin{aligned} \widetilde{L}^{ca} &= \widetilde{\mathrm{CA}}^{\circ} + \widetilde{\mathcal{C}}^{\circ} + \widetilde{\mathcal{F}}^{\circ} + \widetilde{\mathcal{H}} \\ \widetilde{L} &= \widetilde{\mathrm{CA}}^{\circ} + \widetilde{\mathcal{C}}^{\circ} + \widetilde{\mathcal{F}}^{\circ} + \widetilde{\mathcal{H}} + (\widetilde{L}^{cl})^{\circ}, \end{aligned} \tag{16}$$

universite PARIS-SACLAY

Lemma 2

Shareholder cumulative discounted dividends are given by

$$-\widetilde{L} - \widetilde{\mathrm{KVA}}^{\circ}. \tag{17}$$

We emphasize that, in our model, negative dividends are possible. They are interpreted as recapitalisation, i.e. equity dilution.

- All our XVA processes are sought for in a suitable Hilbert space $\mathcal{S}_2$ of square integrable $\mathbb{G}$ adapted processes containing the null process, defined until time $\bar{\tau}$
  - $(\mathbb{G}, \mathbb{Q})$ valuation is never needed beyond that point.
- We denote by $\mathcal{S}_2^{\circ}$ the corresponding subspace of processes Y without jump at τ and such that $Y_T = 0$ on $\{T < \tau\}$.

Outline

Clean valuation and mark-to-market of the portfolio

Definition 2

- Given an $\mathbb{F}$ adapted cumulative cash flow stream $\mathcal{D}$, the OIS discounted $(\mathbb{F}, \mathbb{P})$ value process of $\mathcal{D}$ is the $(\mathbb{F}, \mathbb{P})$ conditional expectation process of the future OIS discounted cash flows in $\mathcal{D}$.
- By clean valuation of a contract (or portfolio) with $\mathbb{F}$ adapted contractually promised cash flow stream $\mathcal{D}$, we mean the $(\mathbb{F}, \mathbb{P})$ value process of $\mathcal{D}$.

Proposition 1

- *Clean valuation is additive over contracts, i.e. the mark-to-market of a portfolio of contracts is the sum of the mark-to-markets of the individual contracts.*
- *Clean valuation is also intrinsic to the contracts themselves. In particular, it is independent of the involved parties and of their collateralization, funding and hedging policies.*

Proof. The promised cash flows of a portfolio are the sum of the promised cash flows of the contracts. Hence the result follows by linearity of the cash flow $(\mathbb{F}, \mathbb{P})$ valuation rule of Definition 2.

Consistently with the $(\mathbb{F}, \mathbb{P})$ local martingale condition on the trading loss process of the clean desks:

Definition 3

By mark-to-market MtM of the (cleaned or client) portfolio, we mean the clean valuation of the contracts not yet liquidated in the portfolio (i.e. ignoring counteparty risk for the future).

Valuation of the Funding and Hedging Assets

Definition 4

Given a $\mathbb{G}$ adapted cumulative cash flow stream $\mathcal{D}$, the OIS discounted $(\mathbb{G}, \mathbb{Q})$ value process of $\mathcal{D}$ is the $(\mathbb{G}, \mathbb{Q})$ conditional expectation process of the future OIS discounted cash flows in $\mathcal{D}$.

Assumption 7

The funding costs of the CA desk are of the form

$$(-\text{OIS accrual of the RC account}) + \mathcal{F}, \tag{18}$$

for some $(\mathbb{G}, \mathbb{Q})$ martingale $\mathcal{F}$ starting from 0 with nondecreasing $\mathcal{F}^{\circ}$ and $\mathcal{F}^{\bullet}$ components.

- The first term in (18) is the funding benefit to which funding would boil down if risk-free funding was available to the bank.
- The rationale underlying Assumption 7 is that funding is implemented in practice as the stochastic integral of predictable hedging ratios against traded funding assets.

- Under the cash flow $(\mathbb{G}, \mathbb{Q})$ valuation rule of Definition 4, the value process of each of these assets is a martingale modulo risk-free accrual.
- Therefore the funding costs of the bank accumulate into a $(\mathbb{G}, \mathbb{Q})$ martingale $\mathcal{F}$, coming on top of a risk-free accrual (actual benefit, i.e. negative cost) of the RC cash account of the CA desk.

- The assumption that $\mathcal{F}^\circ$ is nondecreasing rules out models where the bank can invest (not only borrow) at his unsecured borrowing spread over OIS, because, as a consequence on $\mathcal{F}^\bullet$ through the martingale condition on the process $\mathcal{F}$ as a whole, this would imply that the bank can hedge its own jump-to-default exposure.
- That is, we assume only windfall at bank own default, no shortfall
    - cf. the detailed discussion of BK papers later in these slides.

- In view of Assumption 6, this also insures some kind of orthogonality between the risky funding and hedging loss martingales $\mathcal{F}$ and $\mathcal{H}$, so that $\mathcal{F}$ and $\mathcal{H}$ are nonsubstituable to each other (and the bank cannot manipulate by using one for the other).

Example 2

- Let

$$dB_t = r_t B_t dt$$
$$dD_t = (r_t + \lambda_t) D_t dt + (1 - R) D_{t-} dJ_t = r_t D_t dt + D_{t-}\big(\lambda_t dt + (1 - R) d$$

represent the risk-free OIS deposit asset and a risky bond issued by the bank for its investing and unsecured borrowing purposes.

  - λ represents the unsecured funding spread of the bank and R is the corresponding recovery coefficient, taken as an exogenous constant.

Example 2 (Cont'd)

- The risk-neutral martingale condition that applies to (βD) under our standing valuation framework implies that $\lambda = (1 - R)\gamma$, hence

$$\lambda_t dt + (1 - R)\, dJ_t = (1 - R) d\mu_t,$$

where $d\mu_t = \gamma dt + dJ_t$ is the $(\mathbb{G}, \mathbb{Q})$ compensated jump-to-default martingale of the bank

Example 2 (Cont'd)

- We assume all re-hypothecable collateral and we denote by Q the amount of collateral posted by the CA desk to the clean desks net of the amount received by the CA desk from the clients.

Example 2 (Cont'd)

- The funding policy of the CA desk is represented by a splitting of the amount CA_t on the RC account of the bank as

$$\mathrm{CA}_t = \underbrace{Q_t}_{\text{Collateral remunerated OIS}} + \underbrace{(\mathrm{CA}_t - Q_t)^+}_{\text{Cash in excess invested at the risk-free rate}} - \underbrace{(\mathrm{CA}_t - Q_t)^-}_{\text{Cash needed unsecurely funded}}$$

$$= \underbrace{\left(Q_t + (\mathrm{CA}_t - Q_t)^+\right)}_{\text{Invested at the risk-free rate as } \nu_t B_t} - \underbrace{(\mathrm{CA}_t - Q_t)^-}_{\text{Unsecurely funded as } \eta_t D}$$

Example 2 (Cont'd)

- A standard continuous-time self-financing equation expressing the conservation of cash flows at the level of the bank as a whole yields

$$\begin{aligned} d\left(\nu_t B_t - \eta_t D_t\right) &= \nu_t dB_t - \eta_{t-} dD_t \\ &= \nu_t r_t B_t dt - \eta_t (r_t + \lambda_t) D_t dt - (1-R)\eta_{\tau-} D_{\tau-} dJ_t \\ &= r_t \mathrm{CA}_t dt - (1-R)\eta_{t-} D_{t-} d\mu_t \,, \;\; 0 \le t \le \bar{\tau} \end{aligned} \tag{20}$$

  - A left-limit in time is required in η because D jumps at time τ, so that the process η, which is defined implicitly through $(\mathrm{CA} - Q)^{-}/D$ in (19), is not predictable.

Example 2 (Cont'd)

- Equivalently viewed in terms of costs, i.e. flipping signs in the above, we obtain

$$- d\left(\nu_t B_t - \eta_t D_t\right) = -r_t \mathrm{CA}_t dt + d\mathcal{F}_t,$$

where $d\mathcal{F}_t = (1-R)(Q_{t-} - \mathrm{CA}_{t-})^+ d\mu_t$, which is in line with Assumption 7

Regarding now hedging losses:

Assumption 8

The hedging loss $\mathcal{H} = \mathcal{H}^{\circ}$ of the CA desk, including the cost of setting the hedge, is a $(\mathbb{G}, \mathbb{Q})$ local martingale starting from 0.

- The rationale here is that hedging gains or losses arise in practice as the stochastic integral of predictable hedging ratios against wealth processes of traded hedging assets.
- Note that we are considering wealth processes inclusive of the associated funding costs here, which corresponds to the most common situation of hedges that are either swapped or traded through a repo market, without upfront payment.

- Under the cash flow $(\mathbb{G}, \mathbb{Q})$ valuation rule of Definition 4, each hedging asset is valued as risk-free discounted expectation of its future cash flows.
- Hence the wealth processes related to long positions in any of the hedging assets are $(\mathbb{G}, \mathbb{Q})$ local martingales, as are stochastic integrals against them.

Example 3

Assuming the CA hedge implemented through a repo market on a Black-Scholes stock S with volatility σ, then, supposing no dividends and no repo basis on S:

$$d\mathcal{H}_t = -\zeta_t\big(dS - rS_t dt\big) = -\zeta_t \sigma S_t dW_t, \tag{21}$$

where W is the $(\mathbb{G}, \mathbb{Q})$ Brownian motion driving S and ζ is the hedging ratio used in S.

- The instantaneous cost of funding the hedge is $(\zeta_t r S_t dt)$, which is included in (21).

Remark 2

The valuation impact of a theoretical (but impractical) hedge by the bank of its contra-liabilities will be considered separately in Proposition 3.

As immediate consequences of Assumptions 7 and 8 :

Corollary 3

The processes $\mathcal{F}$ *and* $\mathcal{H} = \mathcal{H}^{\circ}$ *are* $(\mathbb{G}, \mathbb{Q})$ *martingales with zero* $(\mathbb{G}, \mathbb{Q})$ *value.*

Proposition 2

CA, CVA, and FVA are the solutions to the following fixed-point problems, assumed well-posed in $\mathcal{S}_2^\circ$: For $t \leq \bar{\tau}$,

$$\begin{aligned}\widetilde{\mathrm{CA}}_t &= \mathbb{E}_t\big(\widetilde{\mathcal{C}}^\circ_{\bar{\tau}} + \widetilde{\mathcal{F}}^\circ_{\bar{\tau}} - \widetilde{\mathcal{C}}^\circ_t - \widetilde{\mathcal{F}}^\circ_t + \mathbb{1}_{\{\tau<T\}}\widetilde{\mathrm{CA}}^\circ_\tau\big)\\ &= \mathbb{E}_t\big(\widetilde{\mathcal{C}}^\circ_{\bar{\tau}} + \widetilde{\mathcal{F}}^\circ_{\bar{\tau}} - \widetilde{\mathcal{C}}^\circ_t - \widetilde{\mathcal{F}}^\circ_t + \mathbb{1}_{\{\tau<T\}}\widetilde{\mathrm{RC}}^\circ_\tau\big),\end{aligned} \tag{22}$$

by (13), and

$$\widetilde{\mathrm{CVA}}_t = \mathbb{E}_t\big(\widetilde{\mathcal{C}}^\circ_{\bar{\tau}} - \widetilde{\mathcal{C}}^\circ_t + \mathbb{1}_{\{\tau<T\}}\widetilde{\mathrm{CVA}}^\circ_\tau\big), \tag{23}$$

$$\widetilde{\mathrm{FVA}}_t = \mathbb{E}_t\big(\widetilde{\mathcal{F}}^\circ_{\bar{\tau}} - \widetilde{\mathcal{F}}^\circ_t + \mathbb{1}_{\{\tau<T\}}\widetilde{\mathrm{FVA}}^\circ_\tau\big). \tag{24}$$

Proof. By application of the $(\mathbb{G}, \mathbb{Q})$ martingale condition on $\widetilde{L}^{ca}$ as per (16), combined with Corollary 3.

Remark 3

- The industry terminology tends to distinguish an FVA, in the technical sense of the cost of funding cash collateral for variation margin, from an MVA, defined as the cost of funding segregated collateral posted as initial margin (see Albanese et al. (2017)).
- The academic literature, as in these slides, tends to merge the two in an overall FVA meant in the broad sense of the cost of funding the derivative trading strategy of the bank.

Definition 2 (Cont'd)

By contra-liabilities value process CL, we mean

$$\mathrm{CL} = \mathrm{DVA} + \mathrm{FDA} + \mathrm{CVA}^{\mathrm{CL}} + \mathrm{FVA}^{\mathrm{CL}}, \tag{25}$$

where:

- The DVA (debt valuation adjustment) is the $(\mathbb{G}, \mathbb{Q})$ value of $\mathcal{C}^{\bullet}$,
- The FDA (funding debt adjustment) is the $(\mathbb{G}, \mathbb{Q})$ value of $\mathcal{F}^{\bullet}$, and
- $\mathrm{CVA}^{\mathrm{CL}}$ and $\mathrm{FVA}^{\mathrm{CL}}$ are the $(\mathbb{G}, \mathbb{Q})$ values of terminal cash flows $\mathbb{1}_{\{\tau<T\}}\mathrm{CVA}^{\circ}_{\tau}$ and $\mathbb{1}_{\{\tau<T\}}\mathrm{FVA}^{\circ}_{\tau}$ at time $\bar{\tau}$.

Financial interpretation

- The DVA is the value that the bank clients lose due to the possible default of the bank in the future.
- The FDA is the value of the amount of its funding debt that the bank fails to reimburse if it defaults.
- $\mathrm{CVA}^{\mathrm{CL}}$ and $\mathrm{FVA}^{\mathrm{CL}}$ are contra-liability components of the CVA and the FVA, valuing the residual amounts $\mathbb{1}_{\{\tau<T\}}\mathrm{CVA}^{\circ}_{\tau}$ and $\mathbb{1}_{\{\tau<T\}}\mathrm{FVA}^{\circ}_{\tau}$, summing up to $\mathbb{1}_{\{\tau<T\}}\mathrm{CA}^{\circ}_{\tau} = \mathbb{1}_{\{\tau<T\}}\mathrm{RC}^{\circ}_{\tau}$ (cf. (13)), which are transferred from the RC account to bank creditors at time τ.

- Let us now assume, for the sake of the argument, that the bank would be able to hedge its own jump-to-default risk by selling on the financial markets a contract paying CL_τ at time τ (e.g. through repurchasing of its own bond as contemplated in Burgard and Kjaer (2011a, 2011c)).
- Let CR denote the modified charge to be passed to the client for making the trading loss and profit of the bank a $(\mathbb{G}, \mathbb{Q})$ martingale, assuming this hedge in place.

Proposition 3

We have

$$\mathrm{CR} = \mathrm{CA} - \mathrm{CL}, \tag{26}$$

which is also the $(\mathbb{G}, \mathbb{Q})$ *value of* $\mathcal{C}$ *(or of* $(\mathcal{C} + \mathcal{F} + \mathcal{H})$*).*

Corollary 4

CL *is interpreted as the wealth transfer triggered by the deals from shareholders to creditors, due to the inability of the bank to hedge its own jump-to-default exposure.*

Outline

- Not only a bank cannot hedge its own jump-to-default: It cannot replicate its counterparty default losses either.
- An XVA add-on defined by $\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA}$ ensures that the trading loss L of the bank is zero in expectation.
- But the impossibility of replicating counterparty default losses implies that the trading of the bank generates a non-vanishing loss-and-profit process L.
- Then the regulator comes and requires that capital be set at risk by the shareholders, which therefore require a risk premium.

- Valuation is risk-neutral with respect to the stochastic bases $(\mathbb{F}, \mathbb{P})$ or $(\mathbb{G}, \mathbb{Q})$.
- Economic capital and KVA assess risk and its cost, which refer to the historical probability measure.

- In our setup, the duality of perspective of the clean vs. CA desks, on pricing as reflected by Assumption 5, also applies to risk measurement.
  - Capital calculations are always made "on a going concern", i.e. assuming that the bank is alive, and therefore with respect to the reference filtration $\mathbb{F}$.
  - Instead, cost of capital calculations are made by the management of the bank using the filtration $\mathbb{G}$ including the default of the bank.

- However, in the context of XVA computations entailing projections over decades, the main source of information is market prices of liquid instruments, which allow the bank to calibrate the pricing measure, and there is little of relevance that can be said about the historical probability measure.

- Hence, in our model:

Assumption 9

The estimates of the historical probability measure respectively used in economic capital and cost of capital computations coincide with the pricing measures $\mathbb{P}$ and $\mathbb{Q}$.

- Any discrepancy between the historical and risk-neutral measures is left to model risk, meant to be included in an AVA (additional valuation adjustment) in an FRTB terminology.

Economic Capital

- The economic capital (EC) of the bank is its resource devoted to cope with losses beyond their expected levels that are already taken care of by reserve capital (RC).
- Basel II Pillar II defines economic capital as the 99% value-at-risk of the negative of the variation over a one-year period of core equity tier I capital (CET1), the regulatory metric that represents the wealth of the shareholders within the bank.

- Recently, the FRTB required a shift from 99% value-at-risk to 97.5% expected shortfall.
- In our setup, capital depletions correspond to the trading loss process L.

- Accordingly, also accounting for discounting (and recalling that L' is the $\mathbb{F}$ reduction of L):

Definition 5

Our reference definition for the (discounted) economic capital of the bank at time t is the $(\mathcal{F}_t, \mathbb{P})$ conditional 97.5% expected shortfall of $(\widetilde{L}'_{t+1} - \widetilde{L}'_t)$, which we denote by $\widetilde{\mathrm{ES}}_t(L)$.

- Solvency II introduces a further modification of economic capital, which is required to be in excess of the risk margin (RM), i.e. of the KVA (cf. (13)). This modification is considered later.

- For the purpose of economic capital and cost of capital computations, the trading loss process L of the bank can be considered as an exogenous process ($(\mathbb{G}, \mathbb{Q})$ martingale without jump at τ, by Lemma 1).
- Accordingly we just write $\widetilde{\mathrm{ES}}_t$ for $\widetilde{\mathrm{ES}}_t(L)$, and ES_t for $\mathrm{ES}_t(L)$, the undiscounted version of $\widetilde{\mathrm{ES}}_t(L)$.

Lemma 3

ES *is nonnegative.*

KVA as a Risk Premium

- Counterparty default losses, as also funding payments, are materialities for default if not paid, hence the CVA and the FVA are liabilities (or "contra-assets") to shareholders.
- In contrast, KVA payments are at the discretion of the bank management and released to bank shareholders themselves.

Accordingly:

Assumption 10

The risk margin is loss-absorbing, hence part of economic capital.

Corollary 5

Shareholder capital at risk (SCR) *is the difference between the economic capital* (EC) *of the bank and its risk margin* (RM), *i.e.*

$$\mathrm{SCR} = \mathrm{EC} - \mathrm{RM}. \tag{27}$$

Assumption 11

An exogenous and constant hurdle rate h prevails, in the sense that bank shareholders are constantly maintained by the KVA payments on an "efficient frontier" such that, at any time t

$$\text{"Shareholder instantaneous average return}_t = h \times \mathrm{SCR}_t.\text{"} \tag{28}$$

- In practice the level of compensation required by shareholders on their capital at risk is driven by market considerations. Typically, investors in banks expect a hurdle rate h of about 10% to 12%.
- We assume a constant h for simplicity.
- An endogenous and stochastic hurdle rate would arise in a model of competitive equilibrium, where different banks compete for clients.
    - As opposed to our setup where only one bank is considered.

- In view of Lemma 2 and Corollary 5, where $\mathrm{RM} = \mathrm{KVA}$ holds at all times by (13), and since $\widetilde{L}$ is a $(\mathbb{G}, \mathbb{Q})$ martingale by Lemma 1, the informal statement (28) is formulated in mathematical terms by the requirement that

$$
\begin{aligned}
&(-\widetilde{\mathrm{KVA}}^{\circ}) \text{ has a } (\mathbb{G}, \mathbb{Q}) \text{ drift given as} \\
&\text{the time-integrated process } h\,(\widetilde{\mathrm{EC}} - \widetilde{\mathrm{KVA}}),
\end{aligned}
\tag{29}
$$

  assumed to define a unique KVA process in $\mathcal{S}_2^{\circ}$
  - This includes that the KVA process is defined until $\bar{\tau}$ and without jump at τ.

- However, the KVA equation (29) is only preliminary if EC there is just meant as ES, which would then be forgetful of a consistency condition $\mathrm{SCR} \geq 0$.
- This is fixed in the next section by pushing EC above ES until the constraint is satisfied.

The KVA Constrained Optimization Problem

- Assume that, for any tentative economic capital process C in a suitable Hilbert space $\mathcal{L}_2$ of square integrable processes containing $\mathcal{S}_2$ and the process ES, the equation (cf. (29))

  $(-\widetilde{K}^{\circ})$ has a $(\mathbb{G}, \mathbb{Q})$ drift given as the time-integrated process $h(\widetilde{C} - \widetilde{K})$

  defines a unique process $K = K(C)$ in $\mathcal{S}_2^{\circ}$.

université PARIS-SACLAY

Definition 6

The set of admissible economic capital processes is defined as

$$\mathcal{C} = \{C \in \mathcal{L}_2;\, C \geq \max\big(K(C), \mathrm{ES}\big)\}, \tag{31}$$

where (b) $C \geq \mathrm{ES}$ is the risk acceptability condition and (a) $C \geq K(C)$ is the self-consistency condition.

The KVA Constrained Optimization Problem

- In view of (30) and (31), the natural guess for the smallest and cheapest admissible economic capital process is then

$$\mathrm{EC} = \max(\mathrm{ES}, \mathrm{KVA}), \tag{32}$$

for a process KVA in $\mathcal{S}_2^\circ$ such that

$$\begin{aligned}&(-\widetilde{\mathrm{KVA}}^{\circ}) \text{ has a } (\mathbb{G}, \mathbb{Q}) \text{ drift given as the}\\&\text{time-integrated process } h\big(\max(\widetilde{\mathrm{ES}}, \widetilde{\mathrm{KVA}}) - \widetilde{\mathrm{KVA}}\big).\end{aligned} \tag{33}$$

→ The discounted KVA is a $(\mathbb{G}, \mathbb{Q})$ supermartingale.

Remark 4

In the case of perfect clean and CA hedges where the process L (hence L') is constant, then ES vanishes and $\mathrm{KVA} = 0$ obviously solves (33) in $\mathcal{S}_2^\circ$.

Outline

- Given the actual (incremental) derivative porfolio of a bank, the above can be applied to the version of the portfolio that would be run-off by the bank from time 0 onward until its final maturity T.
- The ensuing XVA numbers are interpreted as the amounts CA $= \mathrm{CVA} + \mathrm{FVA}$ to maintain on the reserve capital (RC) account and KVA to maintain on the risk margin (RM) account, which would allow the bank to go into run-off in line with shareholder interest.

- Such a “soft landing option” is key from a regulator point of view, as it guarantees that the bank should not be tempted to go into snowball or Ponzi kind of schemes where always more trades are entered for the sole purpose of funding previously entered ones.
- Moreover, since we rely on a dynamic analysis, this possibility, for a bank respecting the balance conditions (13) for CVA, FVA, and KVA as per (23), (24), and (33), of going run-off in line with shareholder interest, is granted not only from time 0 onward, but from any future time onward, as long as no new deals occur to the bank.

- A new trade has two impacts: it triggers a wealth transfer from shareholders to bondholders and alters the risk profile of the portfolio.
- This is reflected by a jump "$\Delta\cdot$" of the balance sheet, from the balance sheet related to the endowment (pre-trade portfolio) at the time t where the new deal is considered, to the balance sheet related to the portfolio including the new deal at time t (both portfolios being assumed held on a run-off basis).

- Hence the balance conditions (13) and the associated soft landing option of the bank are impaired, unless the missing RC and RM amounts are sourced from the client of the deal in order to restore them.

$\rightarrow$ Assumption 1 implies

$$\Delta\mathrm{RC} = \Delta\mathrm{CA}, \Delta\mathrm{RM} = \Delta\mathrm{KVA},$$

for ΔXVAs computed on an incremental run-off basis based on (22) and (33).

→ The all-inclusive XVA add-on to the entry price for a new deal, called fund transfer pricing (FTP), is

$$\mathrm{FTP} = \Delta\mathrm{CA} + \Delta\mathrm{KVA} = \Delta\mathrm{CVA} + \Delta\mathrm{FVA} + \Delta\mathrm{KVA}. \tag{34}$$

- Obviously, the endowment has a key impact on the FTP of a new trade. For instance, it can happen that a new deal is risk-reducing with respect to the pre-existing portfolio, in which case $\mathrm{FTP} < 0$.

- The preservation of the balance conditions in between and throughout deals yields a sustainable strategy for profits retention, which is already the key principle behind the Eurozone Solvency II insurance regulation.

- From this "soft landing" perspective it is natural to perform the XVA computations under the following assumption, in line with a run-off procedure where market risk is first hedged out, but we conservatively assume no XVA hedge, and the portfolio is then let to amortize until its final maturity T:

Assumption 12

We assume a perfect clean hedge by the clean traders, i.e. $L = L^{ca}$, and no CA hedge, i.e. $\mathcal{H} = 0$.

As it then immediately follows from Lemma 1:

Corollary 6

We have

$$\widetilde{L} = \widetilde{L}^{ca} = \widetilde{\mathrm{CA}} + \widetilde{\mathcal{C}}^{\circ} + \widetilde{\mathcal{F}}^{\circ}. \quad (35)$$

→ The process L that is used as input to capital and KVA computations is the output of the CA computations, making the XVA problem as a whole self-contained.

- cf. Definition 5 and (33), where $\mathrm{ES} = \mathrm{ES}(L)$

Outline

- "Recursive" $(\mathbb{G}, \mathbb{Q})$ XVA equations as per Duffie and Singleton (1999) or Collin-Dufresne, Goldstein, and Hugonnier (2004)
- Solved by reduction to $(\mathbb{F}, \mathbb{P})$, in the present invariance time framework for τ

Assuming that the pre-intensity γ of τ is $\mathbb{F}$ predictable (without loss of generality by reduction), we denote by:

- $\mathcal{S}_2$, the space of càdlàg $\mathbb{G}$ adapted processes Y over $[0, \bar{\tau}]$ such that

$$\mathbb{E}\Big[Y_0^2 + \int_0^T e^{\int_0^s \gamma_u du} \mathbb{1}_{\{s<\tau\}} d(Y_s^*)^2\Big] < \infty, \tag{36}$$

  where $Y_t^* = \sup_{s\in[0,t]} |Y_s|$;
- $\mathcal{S}_2^\circ$, the subspace of the processes Y in $\mathcal{S}_2$ such that Y is without jump at τ on $\{\tau < T\}$ and $Y_T = 0$ on $\{T < \tau\}$;

- $\mathcal{S}_2^\bullet$, the subspace of the processes Y in $\mathcal{S}_2$ such that $Y_{\bar{T}} = 0$;
- $\mathcal{S}_2'$, the space of càdlàg $\mathbb{F}$ adapted processes Y' over $[0, T]$ such that

$$\mathbb{E}'\Big[\sup_{t\in[0,T]} (Y_t')^2\Big] < \infty \tag{37}$$

and $Y_T' = 0$;

- $\mathcal{L}_2$, the space of $\mathbb{G}$ progressively measurable processes X over $[0, T]$ such that

$$\mathbb{E}\Big[\int_0^T e^{\int_0^s \gamma_u du} \mathbb{1}_{\{s<\tau\}} X_s^2 ds\Big] < +\infty; \tag{38}$$

- $\mathcal{L}_2'$, the space of $\mathbb{F}$ progressively measurable processes X' over $[0, T]$ such that

$$\mathbb{E}'\Big[\int_0^T (X_t')^2 dt\Big] < +\infty. \tag{39}$$

KVA in the Case of a Default-Free Bank

- Note that the primary reason for the KVA to exist is the default of the bank clients, as opposed to the default of the bank itself
  - which on the other hand is the key of the contra-liabilities related wealth transfer issue.
- In this part we suppose the bank default free, i.e.

$$\tau = +\infty\,,\;\; (\mathbb{F},\mathbb{P}) = (\mathbb{G},\mathbb{Q}) \text{ and } \gamma = 0.$$

  - This is then extended to the case of a defaultable bank in the next part.

- At that stage in this part we use the "$\cdot'$" notation, not in the sense of reduction (as $\mathbb{F} = \mathbb{G}$), but simply in order to distinguish the equations in this part, where $\mathbb{F} = \mathbb{G}$, from the ones in the next part, where $\mathbb{F} \neq \mathbb{G}$
    - The data of this subsection will then be interpreted a posteriori as the reductions of the corresponding data in the next subsection.

Given $C' \geq \mathrm{ES}$ representing a putative economic capital process for the bank, consider the following BSDEs (cf. (30) and (33) when $\tau = +\infty$):

$$K'_t = \mathbb{E}'_t \int_t^T \big(hC'_s - (r_s + h)K'_s\big)\,ds\,, \quad t \in [0, T], \tag{40}$$

$$\mathrm{KVA}'_t = \mathbb{E}'_t \int_t^T \big(h\max(\mathrm{ES}_s, \mathrm{KVA}'_s) - (r_s + h)\mathrm{KVA}'_s\big)\,ds\,, \quad t \in [0, T], \tag{41}$$

to be solved for respective processes K' and KVA'.

Lemma 4

Assuming that r is bounded from below and that r , C', and ES are in $\mathcal{L}_2'$, then the BSDEs (40) *and* (41) *are well posed in $\mathcal{S}_2'$, where well-posedness includes existence, uniqueness and comparison. We have, for $t \in [0, T]$,*

$$\mathrm{KVA}'_t = h\mathbb{E}'_t \int_t^T e^{-\int_t^s (r_u + h)du} \max(\mathrm{ES}_s, \mathrm{KVA}'_s) ds. \quad (42)$$

Proof. By application of monotonic coefficient BSDE results (see e.g. Kruse and Popier (2016, Sect. 4)).

universite PARIS-SACLAY

Proposition 4

Assuming that r is bounded from below and that r and ES are in $\mathcal{L}_2'$, we have:

(i) $\mathrm{EC}' = \min \mathcal{C}', \mathrm{KVA}' = \min_{C' \in \mathcal{C}'} K'(C')$;

(ii) *The process KVA' is nonnegative and it is nondecreasing in h.*

Proof. By applications of BSDE comparison theorems (see e.g. Kruse and Popier (2016, Proposition 4)).

universite PARIS-SACLAY

KVA in the Case of a Defaultable Bank

In the case of a defaultable bank, "$\cdot'$" now denoting reduction, then, by the results of Crépey and Song (2017b):

- For any $C \in \mathcal{L}_2$, we have $C' \in \mathcal{L}'_2$ and the $(\mathbb{G}, \mathbb{Q})$ BSDE (30) in $\mathcal{S}_2^\circ$ is equivalent to the $(\mathbb{F}, \mathbb{P})$ BSDE (40) in $\mathcal{S}'_2$ through the correspondence $K = (K')^{\tau-}$ on $[0, \bar{\tau}]$;
- Assuming ES in $\mathcal{L}'_2$, the $(\mathbb{G}, \mathbb{Q})$ KVA BSDE (33) in $\mathcal{S}_2^\circ$ is equivalent to the $(\mathbb{F}, \mathbb{P})$ KVA' BSDE (41) in $\mathcal{S}'_2$ through the correspondence $\mathrm{KVA} = (\mathrm{KVA}')^{\tau-}$ on $[0, \bar{\tau}]$.

Hence, by application of Lemma 4 and Proposition 4 through the above correspondences:

Lemma 5

Assuming that r is bounded from below and that r, C', and ES are in $\mathcal{L}'_2$, then the $(\mathbb{G},\mathbb{Q})$ linear BSDEs (30) for $K = K(C)$ and the $(\mathbb{G},\mathbb{Q})$ KVA BSDE (33) are well posed in $\mathcal{S}_2$, where well-posedness includes existence, uniqueness and comparison.

Theorem 1

Assuming that r is bounded from below and that r and ES are in $\mathcal{L}_2'$:
(i) $\mathrm{EC} = \min \mathcal{C}, \mathrm{KVA} = \min_{C \in \mathcal{C}} K(C)$;
(ii) *The process KVA is nonnegative and it is nondecreasing in h.*

Contra-assets and Contra-liabilities

- The counterparty exposure and funding cumulative cash flow streams $Y = \mathcal{C}$ and $\mathcal{F}$ (recall Assumption 12 set $\mathcal{H} = 0$) are given as $\mathbb{G}$ finite variation processes.
- $\mathcal{C}^\circ$ and $\mathcal{F}^\circ$ can be assumed to be $\mathbb{F}$ finite variation processes, without loss of generality by reduction.

- Regarding the funding cash flows, we assume more specifically:

$$d\mathcal{F}_t^{\circ} = f_t(\mathrm{FVA}_t)dt \text{ until } \tau, \tag{43}$$

for some predictable coefficient (random function) f.
- A structure (43) for $\mathcal{F}$ is a slight departure from our abstract setup, where, for simplicity of presentation in a first stage, $\mathcal{F}$ was introduced as an exogenous process.

- But, as already found in the one-period setup or in the continuous-time example 2, the dependence of $\mathcal{F}$ on the FVA is only semi-linear (i.e. f in (43) is Lipschitz or monotonous) in practice.
- Provided the corresponding FVA fixed-point problem is well-posed, one can readily check, by revisiting all the above, that such dependence does not affect any of the qualitative conclusions in the above.

Lemma 6

For $\mathcal{C}$ and $\mathcal{F}$ thus specialized, the CVA *and* FVA *equations* (23) *and* (24) *in $\mathcal{S}_2^\circ$ are equivalent to the following equations in $\mathcal{S}_2'$:*

$$\mathrm{CVA}'_t = \mathbb{E}'_t \int_t^T \beta_t^{-1}\beta_s d\mathcal{C}_s^\circ\,,\ \ t \in [0, T], \tag{44}$$

$$\mathrm{FVA}'_t = \mathbb{E}'_t \int_t^T \beta_t^{-1}\beta_s f_s(\mathrm{FVA}'_s)ds\,,\ \ t \in [0, T], \tag{45}$$

equivalent through the following correspondence:

$$\mathrm{CVA} = (\mathrm{CVA}')^{\tau-} \text{ and } \mathrm{FVA} = (\mathrm{FVA}')^{\tau-} \text{ on } [0, \bar{\tau}]. \tag{46}$$

Proof. By application of the results of Crépey and Song (2017b)

Example 4

In the setup of Example 2, we have, for $0 \leq t \leq \bar{\tau}$:

$$\begin{aligned} d\mathcal{F}_t^{\circ} &= \lambda_t (Q_t - \mathrm{CA}_t)^+ dt \\ d\mathcal{F}_t^{\bullet} &= (1 - R)(Q_{t-} - \mathrm{CA}_{t-})^+ (-dJ_t). \end{aligned} \quad (47)$$

Hence $\mathcal{F}^{\circ}$ is of the form (43), for $f_t(y) = \lambda_t (Q_t - \mathrm{CVA}_t - y)^+$, and

$$dL_t = dL_t^{ca} = d\mathrm{CA}_t - r_t \mathrm{CA}_t dt + d\mathcal{C}_t^{\circ} + \lambda_t (Q_t - \mathrm{CA}_t)^+ dt. \quad (48)$$

Example 4 (Cont'd)

- Assume further that the bank portfolio involves a single client with default time denoted by τ_1, that $\mathbb{Q}(\tau_1 = \tau) = 0$, that the liquidation of a defaulted party is instantaneous and that no derivative cash flows are due at the exact times τ and τ_1.
- Let J and J^1, respectively R and R_1, denote the survival indicator processes and the recovery rates of the bank and its client.

Example 4 (Cont'd)

- Then Q is of the form $J^1 Q^1$, where Q^1 is the difference between the mark-to-market P of the variation margin provided by the CA desk to the clean desks and the mark-to-market of the variation margin provided to the CA desk by the client.
- Moreover, for $0 \le t \le \bar{\tau}$,

$$
\begin{aligned}
d\mathcal{C}_t^{\circ} &= (1 - R_1)(Q^1_{\tau_1})^+(-dJ^1_t) \\
d\mathcal{C}_t^{\bullet} &= \mathbb{1}_{\{\tau \le \tau_1\}}(1 - R)(Q^1_{\tau})^-(-dJ_t).
\end{aligned}
\tag{49}
$$

universite PARIS-SACLAY

Proposition 5

In the setup of Example 4, assuming that r is bounded from below and that the processes r, λ, and $\lambda(J^1 Q^1 - \mathrm{CVA}')^+$ are in $\mathcal{L}'_2$, and that CVA' in (51) is in $\mathcal{S}'_2$, then the CVA and FVA equations (23) and (24) are well-posed in $\mathcal{S}^\circ_2$ and we have, for $0 \le t \le \bar{\tau}$:

$$\mathrm{CVA}_t = (\mathrm{CVA}')^{\tau-}_t \text{ and } \mathrm{FVA}_t = (\mathrm{FVA}')^{\tau-}_t, \text{ where for } 0 \le t \le T: \tag{50}$$

$$\mathrm{CVA}'_t = \mathbb{E}'_t\big[\mathbb{1}_{\{t<\tau_1<T\}}\beta_t^{-1}\beta_{\tau_1}(1-R_1)(Q^1_{\tau_1})^+\big]; \tag{51}$$

$$\mathrm{FVA}'_t = \mathbb{E}'_t \int_t^T \beta_t^{-1}\beta_s\lambda_s(J^1_s Q^1_s - \mathrm{CVA}'_s - \mathrm{FVA}'_s)^+ ds, \tag{52}$$

$$dL_t = d\mathrm{CA}_t - r_t\mathrm{CA}_t dt + (1-R_1)(Q^1_{\tau_1})^+(-dJ^1_t) + \lambda_t(J^1_t Q^1_t - \mathrm{CA}_t)^+ dt \tag{53}$$

Proposition 5 (Cont'd)

$$\mathrm{CL}_t = \underbrace{\mathbb{E}_t\left[\mathbb{1}_{\{\tau \le \tau_1 \wedge T\}} \beta_t^{-1} \beta_{\tau_1} (1-R)(Q^1_\tau)^-\right]}_{\mathrm{FTDDVA}_t} \tag{54}$$

$$+ \underbrace{\mathbb{E}_t\left[\beta_\tau / \beta_t \mathbb{1}_{\{t<\tau<T\}} (J^1_{\tau-} Q^1_{\tau-} - \mathrm{CA}_{\tau-})^+\right]}_{\mathrm{FDA}_t} \tag{55}$$

$$+ \underbrace{\mathbb{E}_t\left[\beta_t^{-1} \beta_\tau \mathbb{1}_{\{t<\tau<T\}} \mathrm{CVA}'_{\tau-}\right]}_{\mathrm{CVA}_t^{\mathrm{CL}}} + \underbrace{\mathbb{E}_t\left[\beta_t^{-1} \beta_\tau \mathbb{1}_{\{t<\tau<T\}} \mathrm{FVA}'_{\tau-}\right]}_{\mathrm{FVA}_t^{\mathrm{CL}}} \tag{56}$$

université PARIS-SACLAY

Proposition 5 (Cont'd)

$$\mathrm{CR}_t = \underbrace{\mathbb{E}_t\left[\mathbb{1}_{\{t<\tau_1\leq\tau\wedge T\}}\beta_t^{-1}\beta_{\tau_1}(1-R_1)(Q^1_{\tau_1})^+\right]}_{\mathrm{FTDCVA}_t} - \underbrace{\mathbb{E}_t\left[\mathbb{1}_{\{t<\tau\leq\tau_1\wedge T\}}\beta_{\tau_1}/\beta_t(1-R)(Q^1_{\tau})^-\right]}_{\mathrm{FTDDVA}_t}. \quad (57)$$

Proof. Under the assumptions of the proposition, the $(\mathbb{F}, \mathbb{P})$ FVA' BSDE (45) is a monotonous coefficient BSDE well-posed in $\mathcal{S}_2'$, based on the results of Kruse and Popier (2016, Sect. 4). In view of Lemma 6, this proves the CVA and FVA related statements.
The dynamics (53) for L are obtained by plugging into (48) the first line in (49).
The CL and CR formulas (56) and (57) readily follow from (25), (47) and (49) for CL and Definition 2 and (49) for CR.

- Proposition 5 is easily extended to bilateral trade portfolios with several counterparties.
    - cf. Albanese, Caenazzo, and Crépey (2017) and (64)-(65) below

- Proposition 5 is derived in a pure valuation perspective.
- In most other former XVA references in the literature, XVA equations are based on hedging arguments.
    - Most previous XVA works were not considering KVA yet.
    - Under our approach, the KVA is the risk premium for the market incompleteness related to contra-assets.
    - Hence, for consistency, our KVA treatment requires a pure valuation (as opposed to hedging) view on contra-assets.

- Formula (57) is symmetrical, i.e. consistent with the law of one price, in the sense that $(\mathrm{FTDCVA} - \mathrm{FTDDVA})$ corresponds to the negative of the analogous quantity considered from the point of view of the counterparty.

- It only involves the first-to-default CVAs and DVAs, where the default losses are only considered until the first occurrence of a default of the bank or its counterparty in the deal.
    - This is consistent with the fact that later cash flows will not be paid in principle.

- Since the presence of collateral has a direct reducing impact on FTDCVA/DVA, this formula may give the impression that collateralization achieves a reduction in counterparty risk at no cost to either the bank or the clients.

- However, in the present incomplete market setup, the value CR from the point of view of the bank as a whole ignores the misalignement of interest between the shareholders and the creditors of a bank.

- Proposition 5 analyses the cost of counterparty risk to shareholders (CA) and the wealth transfer (CL) triggered from the shareholders to the creditors by the impossibility for the bank to hedge its own jump-to-default exposure.

- Due to the latter and to the impossibility for the bank to replicate counterparty default losses, these contra-liabilities (CL) as well as the cost of capital (KVA) are material to shareholders and need to be reflected in entry prices on top of the fair valuation (CR) of counterparty risk.

- Only the fluctuations of L matter in economic capital calculations, hence the (unknown) value of L_0 is immaterial in all XVA computations.

Outline

- Even though our setup includes the default of the bank itself, which is the essence of the contra-liabilities related wealth transfer issue, we end up with unilateral CVA, FVA and KVA formulas such as (51), (52) and (41) pricing the related cash flows until the end of times T (as opposed to $\bar{\tau} = \tau \wedge T$).
  - And these equations only involve the original discount factor β, without any credit spread.

université PARIS-SACLAY

- This is indeed what follows from a careful analysis of the wealth transfers involved.
- However this also makes the ensuing XVAs more expensive than the bilateral XVAs that appear in most of the related literature.

- A unilateral CVA is actually required for being in line with the regulatory requirement that reserve capital should not diminish as an effect of the sole deterioration of the bank credit spread.

- But a bilateral FVA already satisfies the regulatory monotonicity requirement
    - Essentially, as, when the bank credit spread deteriorates, the shortest duration of a bilateral FVA is compensated by the higher funding spread.

- And the KVA is not concerned by this requirement.
    - Actually, a unilateral KVA might arguably be unjustified, with regard to the fact that bank insolvency means depletion of the whole economic capital of the bank, which includes the risk margin. Hence the notion of transfer of the residual risk margin to creditors at bank default would be pointless.
    - However, the default of a bank does not mean insolvency, but illiquidity mainly.

Assuming all the risk margin already gone at time $\bar{\tau}$ through an additional model feature, such as an operational loss that would occur at τ and trigger instantaneous depletion of economic capital, would result in the following modified KVA equation in $\mathcal{S}_2^{\bullet}$:

$$\begin{aligned}&(-\widetilde{\mathrm{KVA}}^{\circ}) \text{ has a } (\mathbb{G}, \mathbb{Q}) \text{ drift given as the}\\&\text{time-integrated process } h\big(\max(\widetilde{\mathrm{ES}}, \widetilde{\mathrm{KVA}}) - \widetilde{\mathrm{KVA}}\big),\end{aligned} \tag{58}$$

i.e., in the continuous time setup,

$$\mathrm{KVA}_t = h\mathbb{E}_t \int_t^{\bar{\tau}} e^{-\int_t^s (r_u+h)du} \max(\mathrm{ES}_s, \mathrm{KVA}_s)ds,\ t \in [0, \bar{\tau}], \tag{59}$$

or, in an an equivalent $(\mathbb{F}, \mathbb{P})$ formulation, $\mathrm{KVA} = (\mathrm{KVA}')^{\tau}$ on $[0, \bar{\tau}]$, where (compare with (42), noting in particular the "$+\gamma_u$" in the discount factor in (60))

$$\mathrm{KVA}'_t = h\mathbb{E}'_t \int_t^{T} e^{-\int_t^s (r_u+h+\gamma_u)du} \max(\mathrm{ES}_s, \mathrm{KVA}'_s)ds,\ t \in [0, T]. \tag{60}$$

- A bilateral FVA, which already satisfies the regulatory monotonicity requirement on the related reserve capital, might be advocated as follows.
- Assume for the sake of the argument that the portfolio of the defaulted bank with clients is unwounded with risk-free counterparties, called novators.

- The residual amount of CVA reserve capital is required by the novators to deal with the residual counteparty risk on the deals.
- But the residual amount of FVA reserve capital is useless to the novators.

- In view of this one could decide that, upon bank default, the residual FVA capital reserve flows back into equity capital and not to creditors.
- For formalizing this mathematically, one needs disentangle the CA desk into a CVA desk and an FVA desk, each endowed with their own reserve capital account (and hedge).

- This would result in an FVA equation stated in $\mathcal{S}_2^{\bullet}$ as

$$\widetilde{\mathrm{FVA}}_t = \mathbb{E}_t\big(\widetilde{\mathcal{F}}^{\circ}_{\bar{\tau}} - \widetilde{\mathcal{F}}^{\circ}_t\big)\,,\ \ t \leq \bar{\tau}, \tag{61}$$

instead of the FVA equation (24) in $\mathcal{S}_2^{\circ}$.

- That is, in the continuous-time setup: (compare with (45))

$$\mathrm{FVA}_t = \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1} \beta_s f_s(\mathrm{FVA}_s) ds, \; t \in [0, \bar{\tau}], \tag{62}$$

or, equivalently, $\mathrm{FVA} = (\mathrm{FVA}')^{\tau}$ on $[0, \bar{\tau}]$, where

$$\mathrm{FVA}'_t = \mathbb{E}'_t \int_t^T e^{-\int_t^s (r_u + \gamma_u) du} f_s(\mathrm{FVA}'_s) ds, \; t \in [0, T]. \tag{63}$$

- Note again the blended discount factor in (63), as opposed to the risk-free discount factor β in (45).

Outline

- Next we account for the additional FVA reduction provided by the possibility for a bank to post economic capital, on top of reserve capital already included in the above, as variation margin.
- Note that, in principle, uninvested capital (UC) could be used for VM as well, but since UC is not known and could as well be zero in the future, capital is conservatively taken here as (RC+EC).

- Accounting for the use of EC as VM, the VM funding needs are reduced from $(Q - \mathrm{CA})^+$ to $(Q - \mathrm{EC}(L) - \mathrm{CA})^+$.
- As a consequence, instead of an exogenous CA value process feeding the dynamics for L (cf. e.g. (53)), one obtains a FBSDE system made of a forward SDE for L coupled with a backward SDE for the CA value process.

So, assuming n counterparties with survival indicator processes J^i, hence $Q = \sum J^i Q^i$, using here unilateral CVA vs. bilateral FVA and KVA as a practical trade-off:

$$
\begin{aligned}
&L_0 = z \text{ and, for } t \in (0, \bar{\tau}], \\
&dL_t = d\mathrm{CA}_t + \sum_i (1 - R_i)(Q^i_{\tau_i})^+(-dJ^i_t) \\
&\quad + \Big(\lambda_t \big(\sum_i J^i_t Q^i_t - \mathrm{EC}_t(L) - \mathrm{CA}_t\big)^+ - r_t \mathrm{CA}_t\Big) dt,
\end{aligned}
\tag{64}
$$

where

$$
\begin{aligned}
\mathrm{CA}_t = & \underbrace{\mathbb{E}_t \sum_{t<\tau_i<T} \beta_t^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i})^+}_{\mathrm{CVA}_t} \\
& + \underbrace{\mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s\lambda_s\Big(\sum_i J^i_s Q^i_s - \mathrm{EC}_s(L) - \mathrm{CA}_s\Big)^+ ds}_{\mathrm{FVA}_t}\,, \quad 0 \le t \le \bar{\tau}.
\end{aligned}
\tag{65}
$$

- Unless $\lambda = 0$, nonstandard coupling between L and CA through the term $\mathrm{EC}_t(L)$, which entails the conditional law of the one-year-ahead increments of L.
- Crépey, Élie, Sabbagh, and Song (2017) show that:
  - This FBSDE for L and CA can be decoupled into an anticipated BSDE ("McKean" ABSDE) for the underlying FVA process;
  - Our previous results are still valid provided one replaces $(Q - \mathrm{CA})^+$ by $(Q - \mathrm{EC}(L) - \mathrm{CA})^+$ everywhere.

Outline

- The above XVA approach can be implemented by means of nested Monte Carlo simulations for approximating the loss process L required as input data in the KVA computations. Contra-assets (and contra-liabilities if wished) are computed at the same time.
- Since one of our goals in the numerics is to emphasize the impact on the FVA of the funding sources provided by reserve capital and economic capital, we consider the FBSDE (64)–(65) which accounts for the use of EC (on top of RC) as VM.

- Let

$$\mathrm{FVA}_t^{(0)} = \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1} \beta_s \lambda_s (\sum_i J_s^i Q_s^i)^+ ds,$$

which corresponds to the FVA accounting only for the re-hypothecation of the variation margin received on hedges, but ignores the FVA deductions reflecting the possible use of reserve and economical capital as VM.

Picard iteration

$L^{(0)} = z$, $\mathrm{FVA}^{(0)}$ as above, $\mathrm{CA}^{(0)} = \mathrm{CVA} + \mathrm{FVA}^{(0)}$ and, for $k \geq 1$,

$$\begin{aligned} &L_0^{(k)} = z \text{ and, for } t \in (0, \bar{\tau}], \\ &dL_t^{(k)} = d\mathrm{CA}_t^{(k-1)} - r_t \mathrm{CA}_t^{(k-1)} dt + \sum_i (1 - R_i)(Q^i_{\tau_i})^+ (-dJ^i_t) \\ &+ \lambda_t \Big(\sum_i J^i_t Q^i_t - \max\big(\mathrm{ES}_t(L^{(k-1)}), \mathrm{KVA}_t^{(k-1)}\big) - \mathrm{CA}_t^{(k-1)} \Big)^+ dt, \end{aligned} \tag{66}$$

$$\mathrm{CA}_t^{(k)} = \mathrm{CVA}_t + \mathrm{FVA}_t^{(k)} \text{ where } \mathrm{FVA}_t^{(k)} =$$

$$\mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s\lambda_s \Big(\sum_i J_s^i Q_s^i - \max\big(\mathrm{ES}_s(L^{(k)}), \mathrm{KVA}_s^{(k-1)}\big) - \mathrm{CA}_s^{(k-1)}\Big)^+$$

$$\mathrm{KVA}_t^{(k)} = h\mathbb{E}_t \int_t^{\bar{\tau}} e^{-\int_t^s (r_u+h)du} \max\big(\mathrm{ES}_s(L^{(k)}), \mathrm{KVA}_s^{(k-1)}\big) ds. \tag{67}$$

- Numerically, one iterates (66)–(67) as many times as is required to reach a fixed point within a preset accuracy.
- In the case studies we considered, one iteration $(k = 1)$ was found sufficient.

- A second iteration did not bring significant change as
  - In (64)-(65) the FVA feeds into economic capital only through FVA volatility and the economic capital feeds into FVA through a capital term which is typically not FVA dominated
  - In (59), in most cases we have that $\mathrm{EC} = \mathrm{ES}$. The inequality only stops holding when the hurdle rate h is very high and the term structure of EC starts very low and has a sharp peak in a few years, which is quite unusual for a portfolio held on a run-off basis, as considered in XVA computations, which tends to amortize in time.

Economic capital projections

- However, going even once through (66)–(67) necessitates the conditional risk measure simulation of $\mathrm{EC}_t(L)$.
- On realistically large portfolios, some approximation is required for the sake of tractability.

- The simulated paths of $L^{(1)}$ are used for inferring a deterministic term structure

$$\mathrm{ES}^{(1)}(t) \approx \mathrm{ES}_t(L^{(1)}) \quad (68)$$

of economic capital, obtained by projecting in time instead of conditioning with respect to $\mathcal{G}_t$ in ES.

Discussion of the economic capital term structure approximation

- If a corporate holds a bank payable, it typically has a desire to close it, receive cash, and restructure the hedge with a par contract (the bank would agree to close the deal as a market maker, charging fees for the new trade).
- Because of this natural selection, a bank is mostly in the receivables in its derivative business with corporates.

Discussion (Cont'd)

- Hence, the tail-fluctuations of its loss process L are mostly driven by the counterparty default events rather than by the volatility of the underlying market exposure.
- Thus, working with a deterministic term structure approximation $\mathrm{ES}_{(1)}(t)$ of economic capital is acceptable.

Discussion (Cont'd)

- If, by exception, the derivative portfolio of a bank is mostly in the payables, then all XVA numbers are small and matter much less anyway

Symmetric FVA

- A similar argument is sometimes used to defend a symmetric FVA (or SFVA) approach, such as, instead of FVA in (65):

$$\mathrm{SFVA_t} = \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1} \beta_s \widetilde{\lambda}_s \Big(\sum_i J_s^i P_s^i \Big) ds\,, \;\; 0 \le t \le \bar{\tau}, \tag{69}$$

  for some VM blended funding spread $\widetilde{\lambda}_t$
  - cf. Piterbarg (2010), Burgard and Kjaer (2013b), and the discussion in Andersen, Duffie, and Song (2017).
- From the FCA/FBA accounting and funds transfer pricing industry standard to an FVA/FDA accounting and pricing framework.

- This corresponds to an FCA/FBA pricing policy, as opposed to our FVA/FDA approach.
- The explicit, linear SFVA formula can be implemented by standard (non-nested) Monte Carlo simulations.

- For a suitably chosen blended spread $\widetilde{\lambda}_t$, the equation yields reasonable results in the case of a typical bank portfolio dominated by unsecured receivables.
- However, in the case of a portfolio dominated by unsecured payables, this equation could yield a negative FVA, i.e. an FVA benefit, proportional to the own credit spread of the bank, which is not acceptable from a regulatory point of view.

- Asymmetric FVA is more rigorous and has been considered in Albanese and Andersen (2014), Albanese, Andersen, and Iabichino (2015), Crépey (2015), Crépey and Song (2016), Brigo and Pallavicini (2014), Bielecki and Rutkowski (2015), and Bichuch, Capponi, and Sturm (2017).
- In this work, we improve upon such asymmetric FVA models by accounting for the funding source provided by economic capital (cf. (65)).

Outline

- We present two XVA case studies on fixed-income and foreign-exchange portfolios. Toward this end we use the market and credit portfolio models of Albanese, Bellaj, Gimonet, and Pietronero (2011) calibrated to the relevant market data.
- We use nested simulation with primary scenarios and secondary scenarios generated under the risk neutral measure calibrated to derivative data using broker datasets for derivative market data.

- All the computations are run using a 4-socket server for Monte Carlo simulations, Nvidia GPUs for algebraic calculations and Global Valuation Esther as simulation software. Using this super-computer and GPU technology the whole calculation takes a few minutes for building the models, followed by a nested simulation time in the order of about an hour for processing a billion scenarios on a real-life bank portfolio.

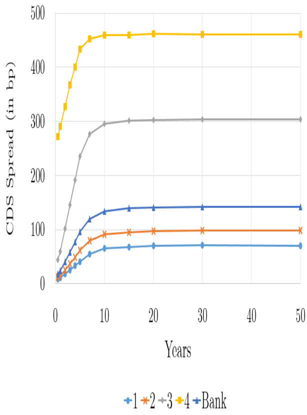
Toy Portfolio

We first consider a portfolio of ten USD currency fixed-income swaps on the date of 11 January 2016 (without initial margins, i.e. for $\mathrm{IM} = 0$).

Toy portfolio of swaps (the nominal of each swap is $\$10^4$)

Mat.	Receiver Rate	Payer Rate	i
10y	Par 6M	LIBOR 3M	3
10y	LIBOR 3M	Par 6M	2
5y	Par 6M	LIBOR 3M	2
5y	LIBOR 3M	Par 6M	3
30y	Par 6M	LIBOR 3M	2
30y	LIBOR 3M	Par 6M	1
2y	Par 6M	LIBOR 3M	1
2y	LIBOR 3M	Par 6M	4
15y	Par 6M	LIBOR 3M	1
15y	LIBOR 3M	Par 6M	4

Credit curves of the bank and its four conterparties



universite PARIS-SACLAY

Introducing financial contracts one after the other in one or the reverse order in a portfolio at time 0 results in the same aggregated incremental FTP amounts for the bank, equal to the "time 0 portfolio FTP", but in different FTPs for each given contract and counterparty.

Toy portfolio. *Left*: XVA values and standard relative errors (SE). *Right*: Respective impacts when Swaps 5 and 9 are added last in the portfolio.

	$Value	SE
$UCVA_0$	471.23	0.46%
$FVA_0^{(0)}$	73.87	1.06%
FVA_0	3.87	4.3%
KVA_0	668.83	N/A
$FTDCVA_0$	372.22	0.46%
$FTDDVA_0$	335.94	0.51%

	Swap 5	Swap 9
$\Delta UCVA_0$	155.46	-27.17
$\Delta FVA_0^{(0)}$	-85.28	-8.81
ΔFVA_0	-80.13	-5.80
ΔKVA_0	127.54	-52.85
$\Delta FTDCVA_0$	98.49	-23.83
$\Delta FTDDVA_0$	122.91	-80.13

universite PARIS-SACLAY

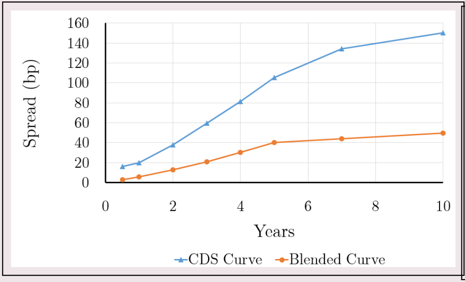
Representative Portfolio

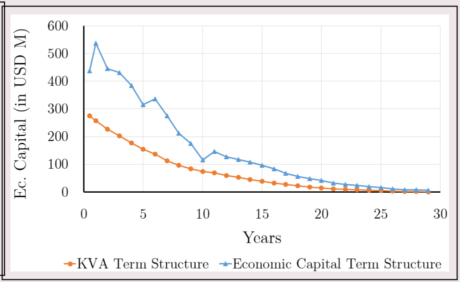
We now consider a representative portfolio with about 2,000 counterparties, 100,000 fixed income trades including swaps, swaptions, FX options, inflation swaps and CDS trades ($IM = 0$).

XVA	\$Value
$UCVA_0$	242 M
$FVA_0^{(0)}$	126 M
FVA_0	62 M
KVA_0	275 M
FTDCVA	194 M
FTDDVA	166 M

universite PARIS-SACLAY

Left: FVA blended funding curve computed from the ground up based on capital projections.
Right: Term structure of economic capital compared with the term structure of KVA.





◂ Return to FVA ◂ Return to KVA ◂ Return to SCR

Outline

Burgard and Kjaer (2011b, 2013a, 2017) repeatedly (and rightfully) say that only pre-default cash-flows matter to shareholders. For instance, quoting the first paragraph in the second reference:

> *"Some authors have considered cases where the post-default cash flows on the funding leg are disregarded but not the ones on the derivative. But it is not clear why some post default cashflows should be disregarded but not others"*,

to which we subscribe fully.

- The introduction of their now classical "(funding) strategy I : semi-replication with no shortfall at own default" (see e.g. (Burgard and Kjaer 2013a, Section 3.2)) seems to be in line with the idea, which we also agree with (see Assumption 7 and the comment following it), that a shortfall of the bank at its own default does not make much sense and should be excluded from a model.

However:

- Being rigorous with the first principle above implies that the valuation jump of the portfolio at the own default of the bank should be disregarded in the shareholder cash flow stream. But their computations do not exclude this cash flow;
- We would not say that they really exclude shortfall at bank own default.

Reviewing the funding strategies in Burgard and Kjaer (2017, Section 4), considering for simplicity the special case with $s_B = 0$ there, i.e. a pure CVA setup:

- Their strategy III, claimed to imply a unilateral CVA as per Albanese and Andersen (2014) (i.e. (50)–(51) as made more precise here), does in fact not.
  - Duly accounting for the transfer of the residual reserve capital from shareholders to creditors at the bank default time τ (cf. (22)), the funding strategy that does so is simply funding and investing at $r = 0$ (having assumed $s_B = 0$).

Reviewing the funding strategies in Burgard and Kjaer (2017, Section 4) (Cont'd):

- Their respective strategies I and II not only do not imply the claimed XVA formulas.
- Besides, they do not satisfy the second part in Assumption 7.
  - Unless in their notation $V \geq 0$, respectively $\widehat{V} \geq 0$, i.e. in our notation $\mathrm{MtM} \geq 0$, respectively $\mathrm{MtM} - \mathrm{CVA} \geq 0$.

- In Green et al. (2014) and as also discussed in some theoretical actuarial literature (see Salzmann and Wüthrich (2010, Section 4.4)), the KVA is treated as a liability.

- Viewing the KVA as a liability, hence non loss-absorbing, results in $\mathrm{EC} = \mathrm{SCR} = \mathrm{ES}$ (as opposed to (27) in our setup), and therefore $h\widetilde{\mathrm{ES}}$ instead of $h(\widetilde{\mathrm{EC}} - \widetilde{\mathrm{KVA}})$ in the KVA equation (29).
- This implies r instead of $(r + h)$ as discount rate in the KVA formula (42) (where KVA' and KVA coincide before τ) or its bilateral analog (59).

- Moreover, if the KVA is viewed as a liability, forward starting one-year-ahead fluctuations of the KVA must be simulated for economic capital calculation. This makes it intractable numerically, unless one switches from economic capital to regulatory capital in the KVA equation.
- Using regulatory instead of economic capital is then motivated by practical considerations but is less self-consistent. It loses the connection, established from shareholder-optimization principles in the above, whereby the correct KVA input is the CA desk loss process $L = L^{ca}$ as per (35).

- It also leads to unachievement of the corresponding XVA theory: Viewing the KVA as a liability means that KVA payments contribute to the trading profit-and-loss of the bank. But dividends are meant to remunerate risk, i.e. unhedged losses. Hence the XVA loop does not close.

- In addition, Green et al. (2014) derive their KVA equation in a replication framework, whereas the main motivation for capital requirements, such as CVA reserve capital to be held against counterparty default losses, is that credit markets are incomplete and hedging is not possible.
- A KVA equation similar to the one in Green et al. (2014) is derived in an expectation setup in Elouerkhaoui (2016).

Outline

Counterparty Risk,

with its funding and capital implications, is at the origin of all XVAs:

CVA Credit valuation adjustment

- The value you lose due to the defaultability of your counterparties

DVA Debit valuation adjustment

- The value your counterparties lose due to your own defaultability
- The symmetric companion of the CVA
- The value you gain due to your own defaultability? (2011 DVA debate)

FVA Funding valuation adjustment

- Cost of funding variation and initial margin: MVA merged with FVA in these slides to spare one "VA"
- But what about the Modigliani-Miller theorem?? (2013 FVA debate)

DVA2 Funding windfall benefit at own default

KVA Cost of capital

- The price for the bank of having to reserve capital at risk (ongoing KVA debate)

CVA−DVA+FVA−DVA2 [+KVA]: The XVA debates

- FVA and DVA2 cash flows NPV-match each other

→ CVA-DVA yields the fair, symmetrical adjustment between two counterparties of equal bargaining power

- But "Contra-liabilities" DVA and DVA2 are only a benefit to the creditors of the bank, whereas only the interest of shareholders matters in bank managerial decisions

→ DVA and DVA2 should be ignored in entry prices

→ CVA+FVA

universite PARIS-SACLAY

KVA and Funds Transfer Price

- Moreover, counterparty default losses cannot be replicated and a bank must reserve shareholder capital to cope with residual risk
- Shareholders that put capital at risk deserve a remuneration at a hurdle rate, which corresponds to the KVA

→ FTP=CVA+FVA+KVA

Connection with the Modigliani and Miller (1958) Theorem

- The Modigliani-Miller theorem includes two key assumptions.
  - One is that, as a consequence of trading, total wealth is conserved.
  - The second assumption is that markets are complete.
- In our setup we keep the wealth conservation hypothesis but we lift the completeness.
- Hence the conclusion of the theorem, according to which the fair valuation of counterparty risk to the bank as a whole should not depend on its funding policy, is preserved.

- However, due to the incompleteness of counterparty risk, derivatives trigger wealths transfers from bank shareholders to creditors
  - The interests of bank shareholders and creditors are not aligned with each other
- Which, in the case of a market maker such as a bank, can only be compensated by add-on to entry prices

More precisely, quoting Villamil (2008):

> *In fact what is currently understood as the Modigliani-Miller Proposition comprises four distinct results from a series of papers (1958, 1961, 1963). The first proposition establishes that under certain conditions, a firms debt-equity ratio does not affect its market value. The second proposition establishes that a firms leverage has no effect on its weighted average cost of capital (i.e., the cost of equity capital is a linear function of the debt-equity ratio). The third proposition establishes that firm market value is independent of its dividend policy. The fourth proposition establishes that equity-holders are indifferent about the firms financial policy.*

- The proof of the fourth proposition is based on the ability of shareholders to redeem all debt of the bank in order to prevent wealth transfers to creditors.

However:

- Redeeming the debt means hedging its own default, which is not possible for a bank.
    - Banks are special firms in that they are intrinsically leveraged and cannot be transformed into a pure equity entity.
    - This is also related to an argument of scale.
        - Banks liabilities are overwhelming with respect to all other wealth numbers.
        - It has been estimated that if all European banks were to be required to have capital equal to a third of liabilities, the total capitalization of banks would be greater than the total capitalization of the entire equity market as we know it today.

Hence:

- Shareholders cannot redeem all debt of the bank.
- The assumption of the fourth proposition of the Modigliani-Miller theorem does not apply to a bank.

- Quoting the conclusion of Modigliani and Miller (1958)

  > *"These and other drastic simplifications have been necessary in order to come to grips with the problem at all. Having served their purpose they can now be relaxed in the direction of greater realism and relevance, a task in which we hope others interested in this area will wish to share."*

- And Miller (1988) in *The Modigliani-Miller Proposition after Thirty Years*

  "Showing what doesn't matter can also show, by implication, what does. "

Outline

XVAs represent a switch of paradigm in derivative management

- From hedging to balance-sheet optimization
- Derivative portfolio optimization for a market maker
  - Fixes prices, not quantities!

Theoretical Issues

- Derivative portfolio optimization for a market maker
- Endogenizing the hurdle rate h by introducing competition between banks
- CCP auction price discovery process
- XVA model risk / uncertainty quantification issue
    - "Prudent valuation" through "Additional valuation adjustment" (AVA)

Algorithmic and computational challenges

- Heavy computations at the portfolio level
- Yet needs accuracy so that incremental XVA computations are not in the numerical noise of the machinery
- Using GPU programming and nested Monte Carlo for portfolio-wide conditional risk measure computations
- Using machine learning techniques for solving the resulting high-dimensional non-convex XVA optimization problems
- Multi-level MC, parareal methods, quasi-regression schemes,..

Albanese, C. and L. Andersen (2014).
Accounting for OTC derivatives: Funding adjustments and the re-hypothecation option.
Working paper available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2482955.

Albanese, C., L. Andersen, and S. Iabichino (2015).
FVA: Accounting and risk management.
Risk Magazine, February 64–68.

Albanese, C., T. Bellaj, G. Gimonet, and G. Pietronero (2011).
Coherent global market simulations and securitization measures for counterparty credit risk.
Quantitative Finance 11(1), 1–20.

Albanese, C., S. Caenazzo, and S. Crépey (2017).
Credit, funding, margin, and capital valuation adjustments for bilateral portfolios.
Probability, Uncertainty and Quantitative Risk 2(7), 26 pages.
Available at http://rdcu.be/tHKo.

Andersen, L., D. Duffie, and Y. Song (2017).
Funding value adjustments.
First revised version available at https://ssrn.com/abstract=2746010.

Bichuch, M., A. Capponi, and S. Sturm (2017).
Arbitrage-free XVA.
Mathematical Finance.
First published online on 18 April 2017 (preprint version available at ssrn.2820257).

Bielecki, T. R. and M. Rutkowski (2015).
Valuation and hedging of contracts with funding costs and collateralization.
SIAM Journal on Financial Mathematics 6, 594–655.

Brigo, D. and A. Pallavicini (2014).
Nonlinear consistent valuation of CCP cleared or CSA bilateral trades with initial margins under credit, funding and wrong-way risks.
Journal of Financial Engineering 1, 1–60.

Burgard, C. and M. Kjaer (2011a).

In the Balance.
Risk, 72–75.

Burgard, C. and M. Kjaer (2011b).
In the balance.
Risk Magazine, October 72–75.

Burgard, C. and M. Kjaer (2011c).
Partial differential equation representations of derivatives with counterparty risk and funding costs.
The Journal of Credit Risk 7, 1–19.

Burgard, C. and M. Kjaer (2013a).
Funding costs, funding strategies.
Risk Magazine, December 82–87.
Preprint version available at https://ssrn.com/abstract=2027195.

Burgard, C. and M. Kjaer (2013b).
Funding Strategies, Funding Costs.
Risk Magazine, December 82–87.

Burgard, C. and M. Kjaer (2017).
Derivatives funding, netting and accounting.
Risk Magazine, March 100–104.
Preprint version available at https://ssrn.com/abstract=2534011.

Castagna, A. and F. Fede (2013).
Measuring and Managing Liquidity Risk.
Wiley.

Collin-Dufresne, P., R. Goldstein, and J. Hugonnier (2004).
A general formula for valuing defaultable securities.
Econometrica 72(5), 1377–1407.

Crépey, S. (2015).
Bilateral counterparty risk under funding constraints. Part I: Pricing, followed by Part II: CVA.
Mathematical Finance 25(1), 1–22 and 23–50.
First published online on 12 December 2012.

Crépey, S., R. Élie, W. Sabbagh, and S. Song (2017).

When capital is a funding source: The XVA Anticipated BSDEs.
Working paper available at https://math.maths.univ-evry.fr/crepey.

Crépey, S. and S. Song (2016).
Counterparty risk and funding: Immersion and beyond.
Finance and Stochastics 20(4), 901–930.

Crépey, S. and S. Song (2017a).
Invariance times.
The Annals of Probability.
Forthcoming.

Crépey, S. and S. Song (2017b).
Invariance times transfer properties and applications.
Working paper available at https://math.maths.univ-evry.fr/crepey.

Duffie, D. (2010).
How big banks fail and what to do about it.
Princeton University Press.

Duffie, D. and M. Huang (1996).

Swap rates and credit quality.
Journal of Finance 51, 921–950.

Duffie, D. and K. Singleton (1999).
Modeling term structures of defaultable bonds.
Review of Financial Studies 12, 687–720.

Elouerkhaoui, Y. (2016).
From FVA to KVA: including cost of capital in derivatives pricing.
Risk Magazine, March 70–75.

Green, A., C. Kenyon, and C. Dennis (2014).
KVA: capital valuation adjustment by replication.
Risk Magazine, December 82–87.
Preprint version "KVA: capital valuation adjustment" available at ssrn.2400324.

Hull, J. and A. White (2012).
The FVA debate, followed by The FVA debate continued.
Risk Magazine, July 83–85 and October 52.

Kruse, T. and A. Popier (2016).

BSDEs with monotone generator driven by Brownian and Poisson noises in a general filtration.
Stochastics: An International Journal of Probability and Stochastic Processes 88(4), 491–539.

Modigliani, F. and M. Miller (1958).
The cost of capital, corporation finance and the theory of investment.
Economic Review 48, 261–297.

Piterbarg, V. (2010).
Funding beyond discounting: collateral agreements and derivatives pricing.
Risk Magazine 2, 97–102.

Salzmann, R. and M. Wüthrich (2010).
Cost-of-capital margin for a general insurance liability runoff.
ASTIN Bulletin 40(2), 415–451.

Villamil, A. (2008).
The Modigliani-Miller theorem.
In The New Palgrave Dictionary of Economics.

Available at http://www.econ.uiuc.edu/ avillami/course-files/PalgraveRev_ModiglianiMiller_Villamil.pdf.